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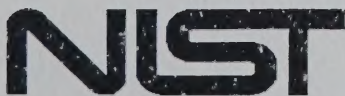
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# **Investigation of a Model for Upward Flame Spread: Transient Ignitor and Burning Rate Effects**

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Cheol Ho Lee



**United States Department of Commerce  
Technology Administration  
National Institute of Standards and Technology**



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Cheol Ho Lee

University of Maryland  
College Park, MD 20742

August 1997



**U.S. Department of Commerce**

William M. Daley, *Secretary*

**Technology Administration**

Gary Bachula, *Acting Under Secretary for Technology*

National Institute of Standards and Technology

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### Notice

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Investigation of a Model for Upward Flame Spread :  
Transient Ignitor and Burning Rate Effects

by

Lee, Cheol Ho

Thesis submitted to the Faculty of the Graduate School  
of The University of Maryland in partial fulfillment  
of the requirements for the degree of  
Master of Science  
1996

Advisory Committee:

Dr. James G. Quintiere, Chairman/Advisor  
Dr. Frederick W. Mowrer  
Dr. Jose L. Torero





## ABSTRACT

Title of Thesis: Investigation of a Model for Upward Flame Spread :  
Transient Ignitor and Burning Rate Effects

Name of degree candidate: Lee, Cheol Ho

Degree and Year: Master of Science in Fire Protection Engineering, 1996

Thesis directed by: Dr. James G. Quintiere, Professor, Department of Fire  
Protection Engineering

Several studies have developed upward flame spread models which use somewhat different features. However, the models have not considered the transient effects of the ignitor and the burning rate. Thus, the objective of this study is to examine a generalized upward flame spread model which includes these effects. We shall compare the results with results from simpler models used in the past in order to examine the importance of the simplifying assumptions. We compare these results using PMMA, and we also include experimental results for comparison. The results of the comparison indicate that flame velocity depends on the thermal properties of a material, the specific model for flame length and transient burning rate, as well as other variables including the heat flux by ignitor and flame itself. The results from the generalized upward flame spread model can provide a prediction of flame velocity, flame and pyrolysis height, burnout time and position, and rate of energy output as a function of time.

## **ACKNOWLEDGEMENTS**

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## NOMENCLATURE

$k$  - thermal conductivity

$\rho$  - density

$c$  - specific heat

$T$  - temperature

$t$  - time

$\tau$  - time

$\Delta t$  - spread time

$x$  - position

$q$  - heat of combustion

$Q$  - power output

$K_f$  - flame height coefficient

$m$  - mass

$\alpha$  - thermal diffusivity

$L$  - heat of gasification

$\Delta H_v$  - heat of vaporization

$\Delta H_c$  - heat of combustion

$l$  - thickness

$n$  - power

$h$  - time step

$\epsilon$  - tolerance for convergence

$\sigma$  - Stefan Boltzmann constant

$\delta$  - thermal penetration depth

$V$  - velocity

$i,j$  - dummy variables

### **Subscripts**

$a$  - ambient

$b$  - burnout

$p$  - pyrolysis

$o$  - initial, ambient

$f$  - flame

$\infty$  - initial, ambient

$ig$  - ignition

$v$  - vaporization

$max$  - maximum

$s$  - steady

$fig$  - flame at ignition

$po$  - pyrolysis at initial

### **Superscripts**

$( \dot{\phantom{x}} )$  - per unit time

$( \phantom{x} )''$  - per unit area





Upward flame spread on vertical surface is a critical aspect of accidental fires because of its inherent high speed and potential consequences of fire growth to surroundings. Most of the principal researchers in the area of fire have devoted significant effort in trying to extend the knowledge on the mechanisms controlling flame spread and mass burning to represent this hazard and attempt to assess the relative contribution for a material. Here this research is interested in the effect of an ignitor, thermal inertia( $kp c$ ) of a material, and burnout during flame spread.

Saito, Quintiere and Williams[1] developed a flame spread model which includes the relationship between flame height, pyrolysis height, and characteristic ignition time. In this model, flame height is controlled by heat released per unit mass of fuel consumed and mass loss rate per unit area, pyrolysis height depends on flame velocity, and characteristic ignition time is dominated by  $kp c$  of a material. They assume that the ignitor effect is zero, which means after ignition, mass loss rate is constant, that is steady burning. In other words, the ignitor effect, burnout effect, and unsteady burning are not included in the solution.

The objective of this research is to develop transient flame spread model which utilizes the numerical solution based on the formulation outlined by Saito, Quintiere and Williams[1]. The model will be dependent on the different  $kp c$  values of a material. The model will be applied to a thermoplastic. Specifically, this research examines the model using polymethylmethacrylate(PMMA), as an example.

The ultimate goal of the research is to examine the flame spread model, which includes the ignitor effect, burnout effect, and transient burning rate model performed by

Hopkins[8], using the data obtained by some researchers[11,12,13] in the program and comparing the results with the experimental results of Orloff, de Ris, and Markstein[11]. The generalized results should provide more accurate predictions in terms of flame spread because it includes transient effects. Using the model we can predict the flame height, pyrolysis height, flame velocity, burnout position and time, total energy release rate at a specific time.



## 2.1 Review of “Upward Turbulent Flame Spread” by Saito, Quintiere, and Williams[1]

### 2.1.1 Description of Spread Mechanisms

Flame Spread occurs as a consequence of heating of the unignited portion of the fuel to a temperature at which vigorous pyrolysis begins. This heating is produced by convective and radiative heat transfer from the flames that bathe the fuel surface. Let  $x$  denote the vertical distance along the fuel surface, with  $x=0$  at the base of the fuel,  $x=x_p$  at the upper edge of the pyrolysis region and  $x=x_f$  at the average height of the visible flame tip, as illustrated in Fig. 2.1. The heat transfer responsible for spread occurs in the region  $x \geq x_p$ . For steady-state burning at the base of a vertical wall, the energy flux  $\dot{q}''$  to the wall has been found experimentally[2] to correlate with  $x/x_f$ , and in a rough first approximation for  $\dot{q}'' = \dot{q}''_0 = \text{constant} \approx 2.5 \text{ W/cm}^2$  for  $0 < x < x_f$  and  $\dot{q}'' = 0$  otherwise, so that  $x_f$  is a good measure of the distance over which the principle heat transfer occurs.

If this rough approximation is employed along with the further assumption that  $x_f - x_p$  remains approximately constant during spread, then the upward spread velocity of pyrolysis front is

$$V_p = 4(\dot{q}''_0)^2 (x_f - x_p) / [\pi k \rho c (T_p - T_a)^2] \quad , \quad (2.1.1)$$

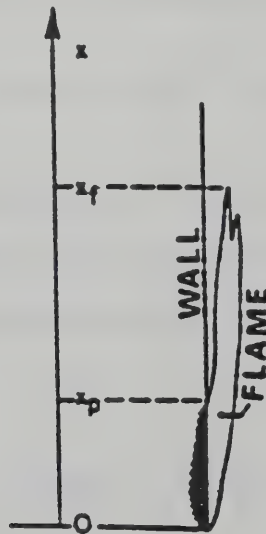
where  $k$ ,  $\rho$ ,  $c$  are the thermal conductivity, density and heat capacity, respectively, of the fuel, and  $T_a$  and  $T_p$  are the ambient and ignition(or pyrolysis) temperatures of the fuel.

Therefore, Equation (2.1.1) can be rewritten as

$$V_p = \frac{x_f - x_p}{\tau} , \quad (2.1.2)$$

where, 
$$\tau = \frac{\pi}{4} k \rho c \left\{ \frac{T_p - T_a}{\dot{q}_0''} \right\}^2 ,$$

the characteristic ignition time  $\tau$  for spread depends only on fuel properties, the ambient temperature and the level of the heat flux to the fuel from flame. As a simplification for describing time-dependent spread, we assume that Eq.(2.1.2) continues to apply with  $x_f - x_p$  variable and that  $\tau$  remains an approximately constant time characteristic of upward spread.



**Fig.2.1** Illustration of the spread model

### 2.1.2 Flame-Height Correlations

Having hypothesized that the correlation of the heat-flux distribution with  $x/x_f$  may lead to Eq.(2.1.2), we need an expression for  $x_f - x_p$  to obtain  $V_p$ . By definition

$$x_p(t) = x_{p0} + \int_0^t V_p(t_p) dt_p, \quad (2.1.3)$$

where  $x_{p0}$  is the value of  $x_p$  at an initial time  $t=0$ , and  $t_p$  is the dummy variable of integration. Flame-height correlations are required for obtaining  $x_f$ . The total rate of energy release per unit length is the sum of

$$\dot{Q}' + q \int_0^x \dot{m}'' dx, \quad (2.1.4)$$

where  $\dot{Q}'$  is the energy release rate per unit length at the base of the wall,  $\dot{m}''$  is the rate of mass loss per unit area of the fuel, and  $q$  is the heat released per unit mass of fuel consumed. Flame-height correlations are of the form

$$x_f = k_f \left[ \dot{Q}' + q \int_0^{x_p} \dot{m}'' dx \right]^n, \quad (2.1.5)$$

where  $k_f$ , flame height coefficient, and  $n$  are constants. The flame height for wall flames is given such that  $k_f = 0.067 (m^5/kw^2)^{1/3}$  and  $n=2/3$ , or approximately  $k_f = 0.01 (m^2/kw)$  and  $n=1$  [2],[3],[4].

## 2.2 Exact Solution for $n=1$

As a basis for describing upward spread we shall assume that the flame spreads after ignition( $\dot{Q}'=0$ ) and the rate of mass loss per unit area( $\dot{m}''$ ) is constant. Following these assumptions and substituting  $n=1$  into Eq.(2.1.5), the flame height can be rewritten as

$$x_f = k_f (q \dot{m}'' x_p) . \quad (2.2.1)$$

Substituting Eq.(2.2.1) into Eq.(2.1.2), Eq.(2.1.2) can be rewritten as

$$V_p = \frac{dx_p}{dt} = \frac{(k_f q \dot{m}'' - 1) x_p}{\tau} . \quad (2.2.2)$$

To derive  $x_p$ , Eq(2.2.2) can be rewritten

$$\frac{dx_p}{x_p} = \frac{(k_f q \dot{m}'' - 1) dt}{\tau} . \quad (2.2.3)$$

Integrating Eq(2.2.3)  $x_p$  can be obtained as

$$x_p = x_{po} e^{(k_f q \dot{m}'' - 1) t / \tau} , \quad (2.2.4)$$

which means  $x_p$  is increases with time( $t$ ). Therefore, substituting Eq.(2.2.4) into

Eq.(2.2.2) the exact solution for  $n=1$  is

$$V_p = \frac{dx_p}{dt} = \frac{(k_f q \dot{m}'' - 1) x_{po} e^{(k_f q \dot{m}'' - 1) t / \tau}}{\tau} . \quad (2.2.5)$$



### 2.3 Exact Solution for $n=2/3$

Following the assumption  $\dot{Q}'=0$  and  $\dot{m}''$  is constant and substituting  $n=2/3$  and Eq.(2.1.5) into Eq(2.1.2),  $V_p$  for  $n=2/3$  can be expressed as

$$V_p = \frac{dx_p}{dt} = \frac{k_f(q\dot{m}'' x_p)^{2/3} - x_p}{\tau} . \quad (2.3.1)$$

Unlike the case of exact solution for  $n=1$ , this case is required some steps to derive  $x_p$  since Eq.(2.3.1) is non-linear.

Let

$$\zeta = x_p^{1/3} , \quad (2.3.2)$$

and differentiate both terms of this, then we have

$$d\zeta = \frac{1}{3} \frac{dx_p}{\zeta^2} , \quad (2.3.3)$$

and

$$dx_p = 3\zeta^2 d\zeta . \quad (2.3.4)$$

Substituting Eq(2.3.4) into Eq.(2.3.1), we have

$$3\tau \frac{d\zeta}{dt} = k_f(q\dot{m}'')^{2/3} - \zeta . \quad (2.3.5)$$

Let

$$\eta = k_f(q\ddot{m})^{2/3} - \zeta, \quad (2.3.6)$$

and differentiate both terms of this, then we have  $d\zeta = -d\eta$ . Substituting these into Eq.(2.3.5), we have

$$3\tau \frac{d\eta}{dt} = \eta, \quad (2.3.7)$$

and

$$\frac{d\eta}{\eta} = -\frac{dt}{3\tau}. \quad (2.3.8)$$

After integration Eq.(2.3.8) we have

$$\frac{\eta}{\eta_o} = e^{(-t/3\tau)}, \quad (2.3.9)$$

and substituting Eq.(2.3.2) into Eq.(2.3.6) and then substituting again Eq.(2.3.6) into Eq.(2.3.9) we have

$$\frac{k_f(q\ddot{m})^{2/3} - x_p^{1/3}}{k_f(q\ddot{m})^{2/3} - x_{po}^{1/3}} = e^{(-t/3\tau)}, \quad (2.3.10)$$

and from Eq.(2.3.10) we have

$$x_p^{1/3} = k_f(q\ddot{m})^{2/3} - (e^{-t/3\tau}) \{ k_f(q\ddot{m})^{2/3} - x_{po}^{1/3} \} . \quad (2.3.11)$$

To show that  $x_p$  is some function of time, Eq.(2.3.11) can be rewritten as

$$x_p^{1/3} = k_f(q\ddot{m})^{2/3} \{ 1 - [1 - x_{po}^{1/3} / k_f(q\ddot{m})^{2/3}] e^{-t/3\tau} \} , \quad (2.3.12)$$

and then ,  $x_p$  is

$$x_p = k_f^3(q\ddot{m})^2 \{ 1 - [1 - x_{po}^{1/3} / k_f(q\ddot{m})^{2/3}] e^{-t/3\tau} \}^3 , \quad (2.3.13)$$

which means  $x_p$  increases with cubic time( $t^3$ ).

Therefore, letting  $x_p = A$  in Eq.(2.3.13) and substituting A into Eq.(2.3.1), the exact

solution for  $n=2/3$  is

$$V_p = \frac{dx_p}{dt} = \frac{k_f(q\ddot{m}A)^{2/3} - A}{\tau} . \quad (2.3.14)$$

### 3.1 Integral Equation Formulation

Since burning rate( $\dot{m}''$ ) is independent of the location of the element, the integral in Eq. (2.1.5) may be written as

$$\int_0^{x_p} \dot{m}'' dx = \int_0^{x_{p0}} \dot{m}'' dx + \int_{x_{p0}}^{x_p} \dot{m}'' dx, \quad (3.1.1)$$

where  $\dot{m}'' = \dot{m}''(x_p(t), t) = \dot{m}''(t)$  at  $x=x_p(t)$ . Since  $0 \leq x \leq x_{p0}$ , Eq.(3.1.1) can be rewritten as

$$\int_0^{x_p} \dot{m}'' dx = \dot{m}''(x_{p0}, t) x_{p0} + \int_{x_{p0}}^{x_p} \dot{m}'' dx \quad (3.1.2)$$

Eq.(3.1.2) shows that burning rate is related to the position of material, and all terms in Eq.(3.1.2) can be changed from the position to time since the position independent with time as shown Fig.3.1. Therefore, Eq.(3.1.2) can be rewritten as

$$\int_{x_{p0}}^{x_p(t)} \dot{m}'' dx = \int_0^t \dot{m}''(t-t_p) \frac{dx_p}{dt} dt_p \quad (3.1.3)$$

Substituting  $(dx_p/dt)_{t=tp} = V_p(t_p)$  into Eq.(3.1.3) and then substituting Eq.(3.1.3) into Eq.(3.1.2), Eq.(3.1.2) becomes



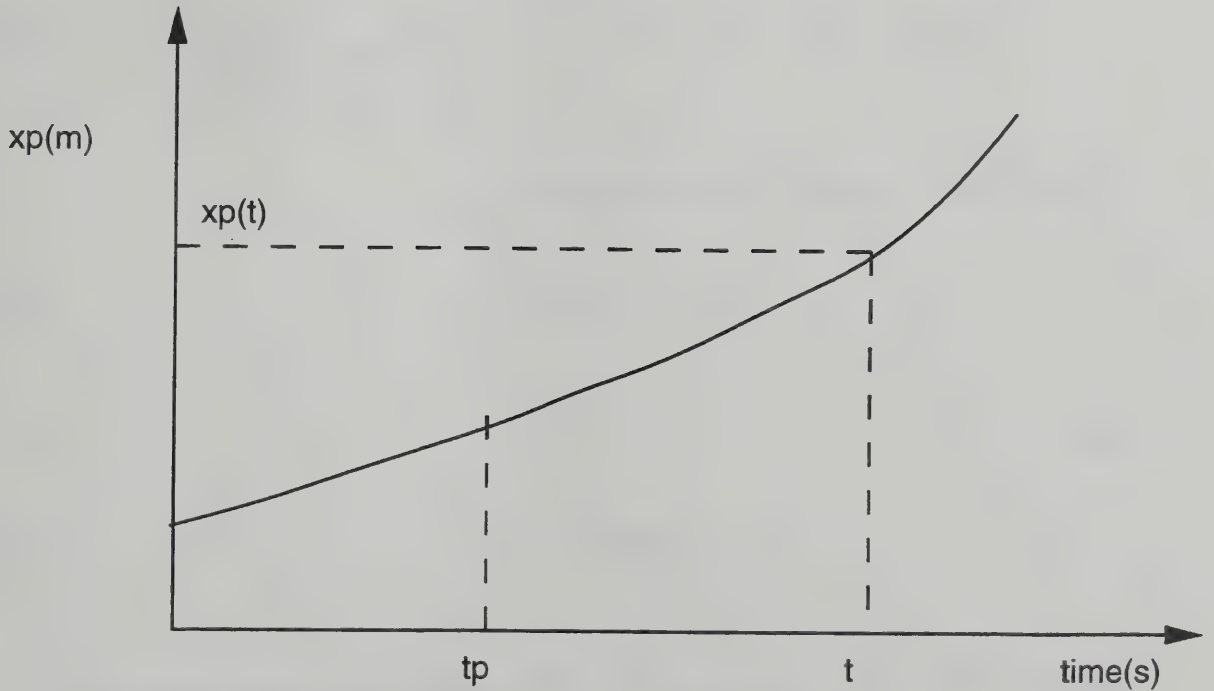
$$\int_0^{x_p} \dot{m}'' dx = \dot{m}''(t) x_{po} + \int_0^t \dot{m}''(t-t_p) V_p(t_p) dt_p \quad (3.1.4)$$

Hence, substituting Eq.(3.1.4) into Eq.(2.1.5), the flame height( $x_f$ ) become

$$x_f = k_f \left[ \dot{Q}' + q \left\{ \dot{m}''(t) x_{po} + \int_0^t \dot{m}''(t-t_p) V_p(t_p) dt_p \right\} \right]^n \quad (3.1.5)$$

Therefore, substituting Eq.(2.1.3) and Eq.(3.1.5) into Eq.(2.2.2), the integral equation for flame spread is

$$V_p(t) = \frac{1}{\tau} \left\{ k_f \left[ \dot{Q}' + q \left\{ \dot{m}''(t) x_{po} + \int_0^t \dot{m}''(t-t_p) V_p(t_p) dt_p \right\} \right]^n - \left[ x_{po} + \int_0^t V_p(t_p) dt_p \right] \right\} \quad (3.1.6)$$



**Fig. 3.1** Illustration of pyrolysis front position response to the time

### 3.2 Numerical Solution

To find the spread velocity with time,  $V_p(t)$ , in Eq.(3.1.6), the integral equation in Eq.(3.1.6) should be solved. This study uses The Trapezoidal Rule[5] to solve the integral equation as a numerical method and an iteration process to find  $V_p(t)$  until convergence is satisfactory.

#### 3.2.1 Approximation Integrals by Trapezoidal Rule

The Trapezoidal Rule for approximating  $\int_a^b f(x) dx$  is given by

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} \left[ f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n) \right] . \quad (3.2.1)$$

To apply Eq.(3.1.6) to the Trapezoidal Rule let  $t_p = t'$ ,

$$I_1(t) = \int_0^t \ddot{m}(t - t') V_p(t') dt' , \quad (3.2.2)$$

and

$$I_2(t) = \int_0^t V_p(t') dt' . \quad (3.2.3)$$

Following  $[n=1 \rightarrow n=n+1]$  and  $[t'_1=0 \rightarrow t'_{n+1}=t_{n+1}=t]$ , Eq.(3.2.2) can be written as

$$I(t_{n+1}) = \int_0^{t_{n+1}} \ddot{m}(t_{n+1} - t') V_p(t') dt'$$

$$= h \left[ \frac{\ddot{m}(t_{n+1} - t_1) V_p(t_1)}{2} + \ddot{m}(t_{n+1} - t_2) V_p(t_2) + \dots + \ddot{m}(t_{n+1} - t_n) V_p(t_n) + \frac{\ddot{m}(t_{n+1} - t_{n+1}) V_p(t_{n+1})}{2} \right], \quad (3.2.4)$$

where  $h = t_{n+1} - t_n$ .

Defining  $\theta$  as

$$\theta_1 = t_{n+1} - t_1 = t_{n+1} - 0 = t_{n+1}$$

$$\theta_2 = t_{n+1} - t_2 = t_{n+1} - h$$

$$\theta_3 = t_{n+1} - t_3 = t_{n+1} - 2h$$

◦

◦

◦

$$\theta_n = t_{n+1} - t_n = h$$

$$\theta_{n+1} = t_{n+1} - t_{n+1} = 0,$$

$$\therefore \theta_i = t_{n+1} - t_i = t_{n+1} - (i-1)h \quad (3.2.5)$$

where  $t_{n+1} = t_1 + (n)h$ , Eq.(3.2.4) can be rewritten as

$$I(t_{1 \ n+1}) = h \left[ \frac{\dot{m}''(\theta_1)V_p(t_1)}{2} + \dot{m}''(\theta_2)V_p(t_2) + \dots + \dot{m}''(\theta_n)V_p(t_n) + \frac{\dot{m}''(\theta_{n+1})V_p(t_{n+1})}{2} \right] \quad (3.2.6)$$

Therefore, Eq.(3.2.6) becomes

$$I(t_{1 \ n+1}) = \frac{h}{2} \sum_{i=1}^n (\dot{m}''_i V_{p \ i} + \dot{m}''_{i+1} V_{p \ i+1}) \quad (3.2.7)$$

where  $\dot{m}''_i = \dot{m}''(\theta_i)$  ,  $\theta_i = t_{n+1} - t_i = t_{n+1} - (i-1)h$  ,

and  $V_{p \ i} = V_p(t_i)$  ,  $t_i = t_1 + (i-1)h$  ,  $t_1 = 0$  (ignition).

Similarly Eq.(3.2.3) can be written following the process of above as

$$I(t_{2 \ n+1}) = \int_0^{t_{n+1}} V_p(t') dt' \\ = \frac{h}{2} \sum_{i=1}^n (V_{p \ i} + V_{p \ i+1}) \quad (3.2.8)$$

Therefore, following the Trapezoidal Rule, the integral equation, Eq.(3.1.6), can be written



as

$$V_p(t) = \frac{1}{\tau} \left\{ k_f \left[ \dot{Q}' + q \left\{ \ddot{m}''(t)x_{po} + I_1(t) \right\} \right]^n - \left[ x_{po} + I_2(t) \right] \right\} . \quad (3.2.9)$$

### 3.2.2 The Solution of the Integral Equation by Iteration

Assuming a new value( $V_p(t)$ ), which is in the Right Hand Side(RHS) in Eq.(3.2.9), to a previous value( $V_p(t-1)$ ), which is gotten from a previous step, we can get the new value( $V_p(t)$ ), which is in the Left Hand Side(LHS) and is not correct value. To find a real new value( $V_p(t)$ ), some examination is needed like

$$\text{Error} = \frac{|V_p(t) - V_p(t-1)|}{V_p(t)} \leq \epsilon , \quad (3.2.10)$$

where  $\epsilon$  is a tolerance. If Error is greater than  $\epsilon$ , let  $V_p(t) = V_p(t-1)$  and then repeat the process until Error less than equal  $\epsilon$ . This will be shown later in computer program. When this condition is satisfied, we can get a new correct value( $V_p(t)$ ).

## CHAPTER 4

## Comparison of Exact Solution and Numerical Solution Using Computer Program

A numerical solution is not exact since the solution comes from integral and difference approximations. We, however, will apply this numerical algorithm to a generalized flame spread model that will be discussed later, therefore; we need to test its accuracy. To do this test, we shall compare the difference of results obtained from the exact and numerical solutions

The exact solutions used for testing are taken from Chapter 2 ; (1)  $x_f \propto x_p$  and (2)  $x_f \propto x_p^{2/3}$ . The exact solutions are given by Equations (2.2.4 )and(2.3.13). In both cases the mass burning rate per unit area is constant and the ignitor effect is zero.

Since flame velocity depends on the differences between flame height( $x_f$ ) and pyrolysis zone( $x_p$ ), we can predict the correlation as shown Figure 4.1. In both case, the velocity eventually becomes zero. In the case of  $n=2/3$  the point, where flame height is equal to pyrolysis( $x_f = x_p$ ), is earlier than the point in  $n=1$ . Therefore, the time in  $n=2/3$ , where flame velocity starts to decrease, is earlier than the time in  $n=1$ . This zero velocity point is an usual feature of both solutions, and may not be physical since  $\tau$  should decrease as the flame gets bigger. In any case they still form good tests for the numerical results.

The numerical algorithm is programmed in Fortran. Also, the variables and data used for programs are in Appendix A. The program is list in Appendix B.

### 4.1 The Variables and Data used for Testing

The material used for this comparison is PMMA and properties of this material are taken from Quintiere and Rhodes's experiment[6]. The energy flux  $\dot{q}''$  was already mentioned in Chapter 2. In steady state, the mass loss rate can be obtained from

$$\dot{m}'' = \frac{\dot{q}''_f - \sigma T_{ig}^4}{L}, \quad (4.1.1)$$

where  $\dot{q}''_f$  is the heat flux from flame, and  $\sigma$  is Stefan Boltzmann constant ( $5.67 \times 10^{-11}$

$\text{kw/m}^2\text{k}^4$ ), and  $L$  is heat of gasfication. The initial pyrolysis zone  $x_{po}$  is selected as 0.3m

and the input data are shown in Appendix A. All of input data used for the exact solutions (for  $n=1$  and  $n=2/3$ ) and the numerical solutions (for  $n=1$  and  $n=2/3$ ) are same.

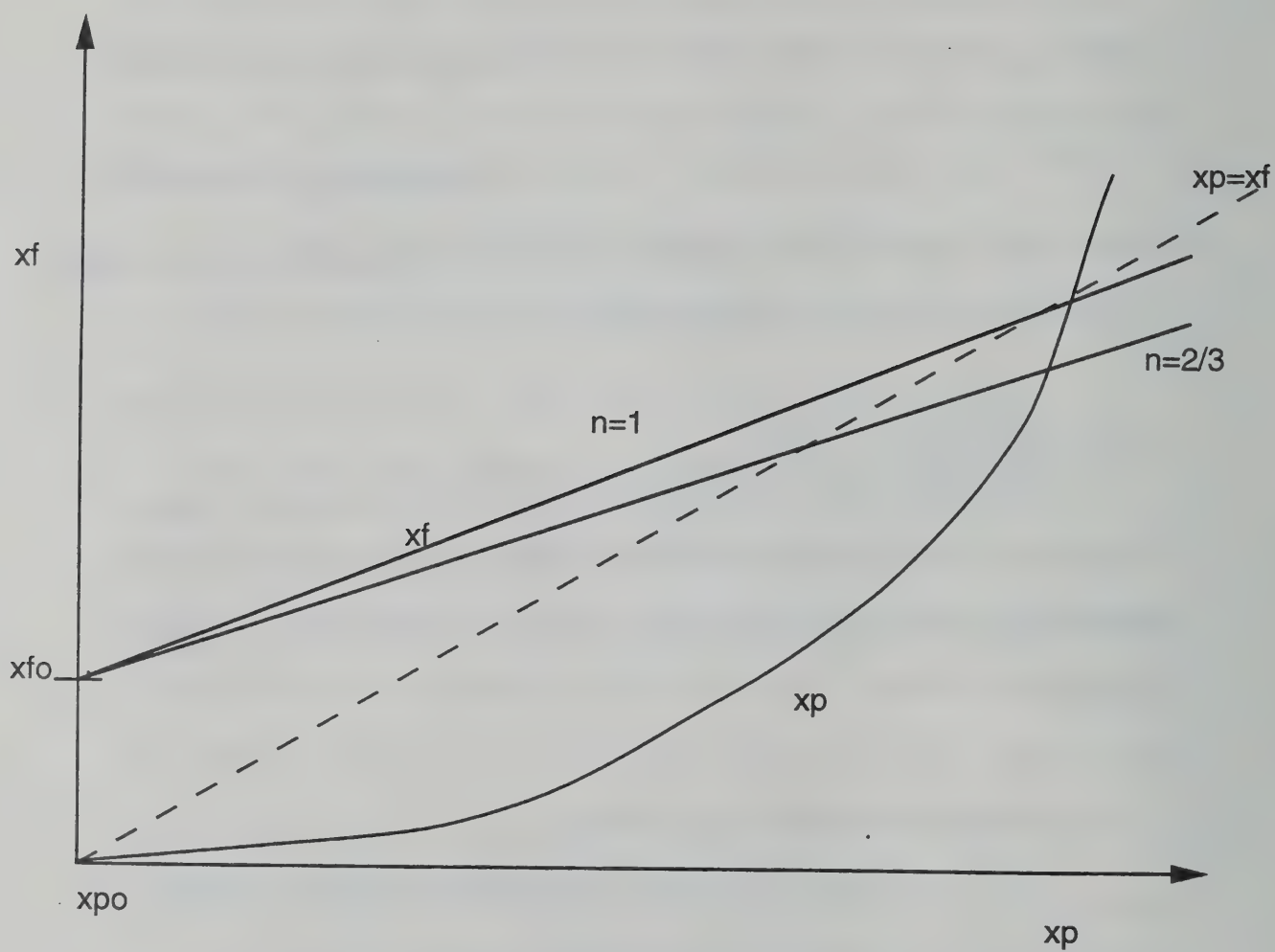
## 4.2 Programs for Testing

Each program can be developed following the process described in Chapter 3 respectively. In the program for the numerical solution, the tolerance(TOL) used for convergence is  $10^{-4}$  and the time step(H) is 1.0 second. These programs are shown in Appendix B.

## 4.3 Comparison of Results for Testing

Since flame velocity( $V_p$ ) is related to flame height( $x_f$ ) and pyrolysis zone( $x_p$ ) as shown Eq.(2.2.2), we just compare results of flame velocity obtained from the calculations.

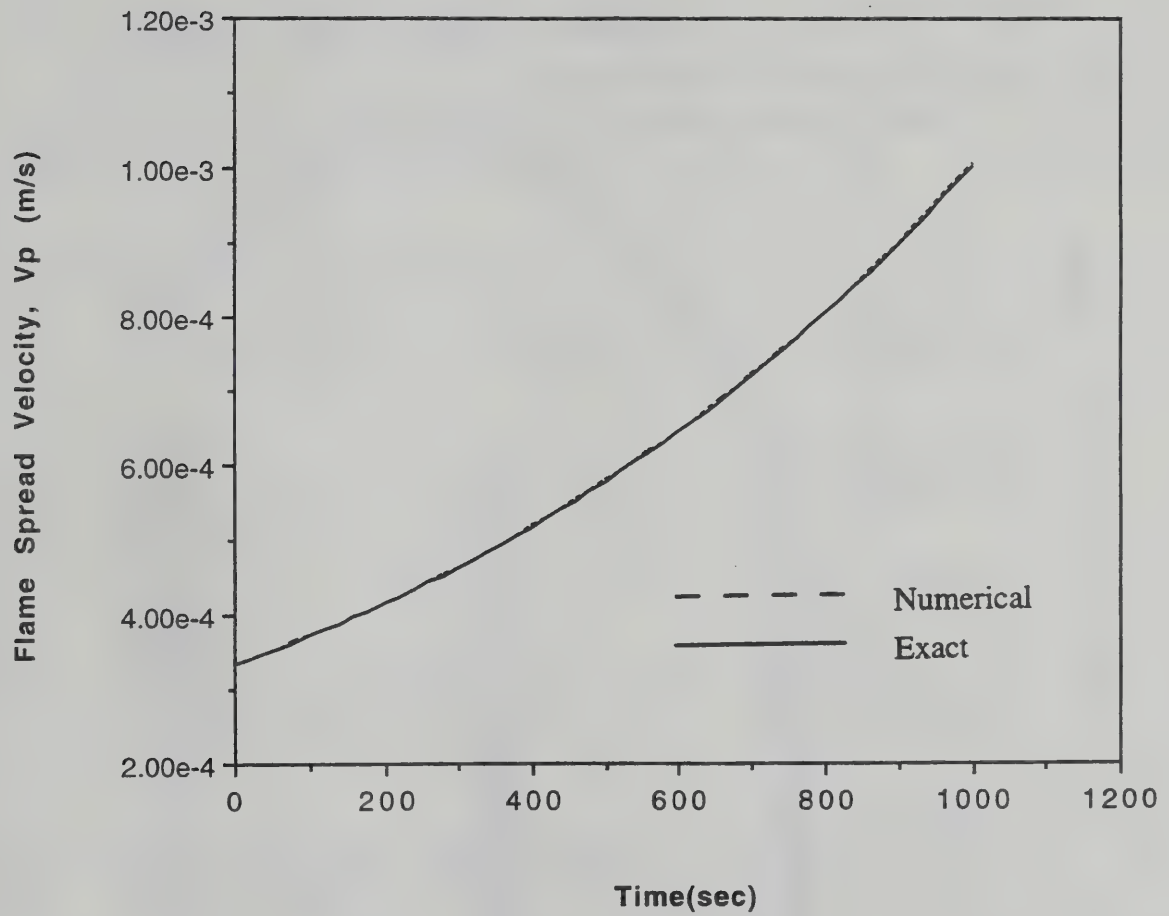
Figure 4.2 and 4.3 show the difference of flame velocity between the exact solution and the numerical solution at  $n=1$  and  $n=2/3$  respectively. The differences are negligible. Therefore, we can say the numerical solution procedure can be used in the generalized flame spread model with expected similar accuracy.



**Figure 4.1** The correlation between flame height and pyrolysis zone dependent on different powers ( $n$ ).

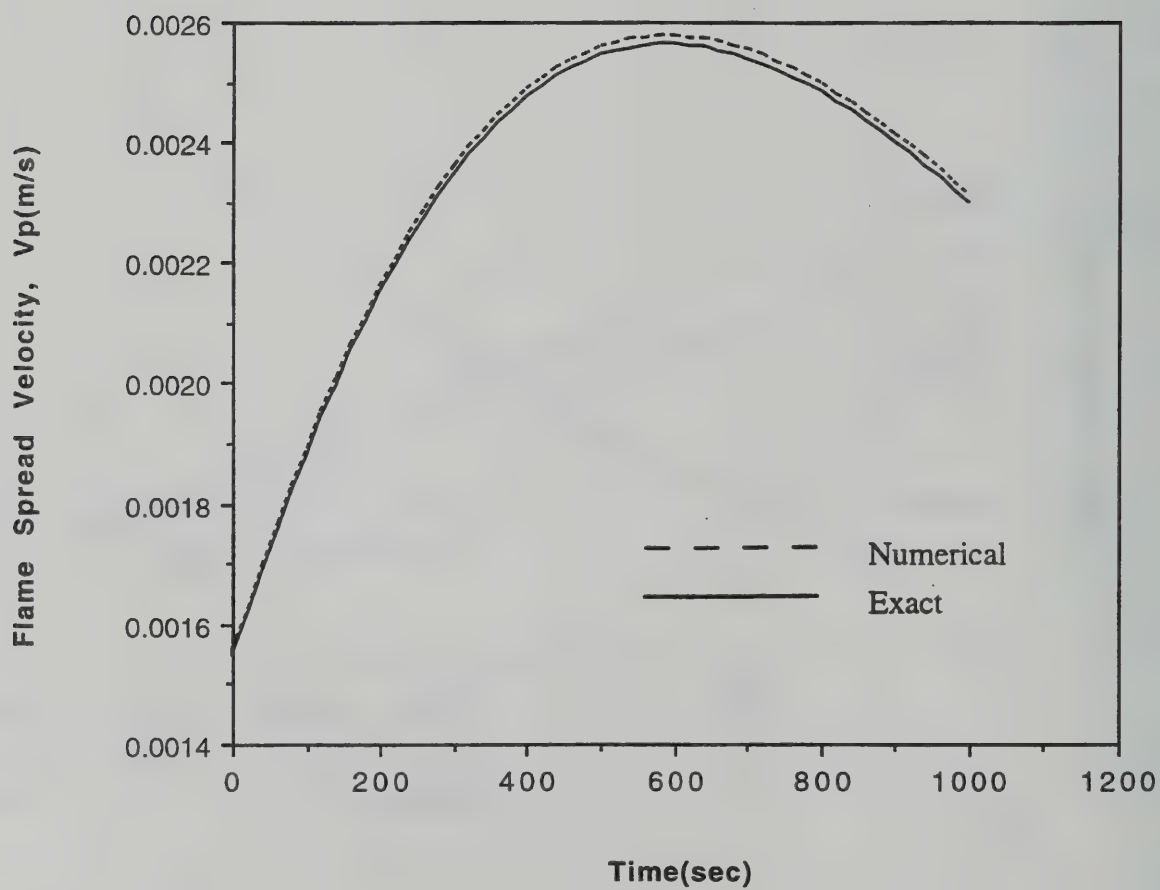


### Upward Turbulent Fire Spread



**Figure 4.2** Comparison of flame spread velocity for PMMA between exact and numerical solution for  $n=1$  as a function of time.

### Upward Turbulent Fire Spread

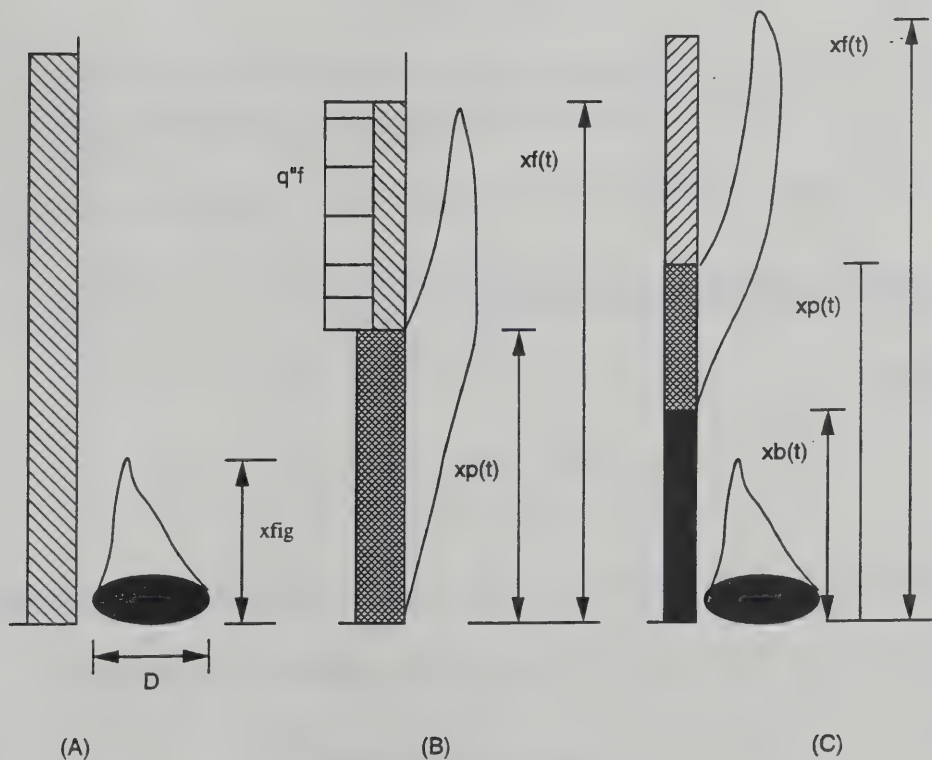


**Figure 4.3** Comparison of flame spread velocity for PMMA between exact and numerical solution for  $n=2/3$  as a function of time.

We have discussed flame height and flame velocity after ignition and under constant mass loss rate. In general, flame height and velocity, however, can be affected by the ignitor, burnout, and transient burning rate. Therefore, we need a general model that includes the effect of an ignitor, burnout, and burning rate to analyze and predict a real fire situation. The model will be described below.

### 5.1 Flame Height Calculations

As shown Figure 5.1 flame spread can be separated with three parts. Figure



**Figure 5.1** Configuration of flame spread, (A)Before Ignition (B) After Ignition (C)After Burnout

(5.1.A), (5.1.B), and (5.1.C) show the flame height effected by before ignition, after ignition and after burnout respectively.

Flame height is solely due to the ignitor before an ignition occurs as shown Figure (5.1.A). Its flame height can be computed or experimentally determined according to its configuration[7]. For example, if its configuration is like a pool fire then

$$x_{fig} = 0.23 \dot{Q}_{ig}^{2/5} - 1.02 D_{ig} , \quad (5.1.1)$$

where  $\dot{Q}_{ig}$  is the ignitor source(kw). If the ignitor is more like a line fire of width W against the wall, then

$$x_{fig} = k_f(\dot{Q}_{ig}/W)^n , \quad (5.1.2)$$

where the correlation between  $k_f$  and  $n$  is the same as before (Eq. 2.1.5).

Figure (5.1.B) shows the flame height after wall ignites due to  $\dot{Q}_{ig}$  and  $\dot{Q}'$  up to burn out of the initial region ignited( $t_{ig} \leq t < t_b(x_{fig})$ ). At this situation flame height becomes

$$x_f(t) = k_f \left[ (\dot{Q}_{ig} / W) + \dot{Q}' \right]^n , \quad (5.1.3)$$

where  $\dot{Q}'$  is the wall contribution. Figure (5.1.C) shows the flame spread after initially ignited burn out( $t \geq t_b(x_{fig})$ ). At this time flame height can be written as

$$x_f(t) = x_b(t) + k_f(\dot{Q}')^n . \quad (5.1.4)$$

We can unify Figure (5.1) A, B, and C by introducing step functions

$$\eta(x) = 1, x \geq 0$$

or

$$\eta(x) = 0, x < 0.$$

Therefore, we can let

$$\dot{Q}_{ig} = \eta(t_b(x_{fig}) - t) \cdot \dot{Q}_{ig}, \quad (5.1.5)$$

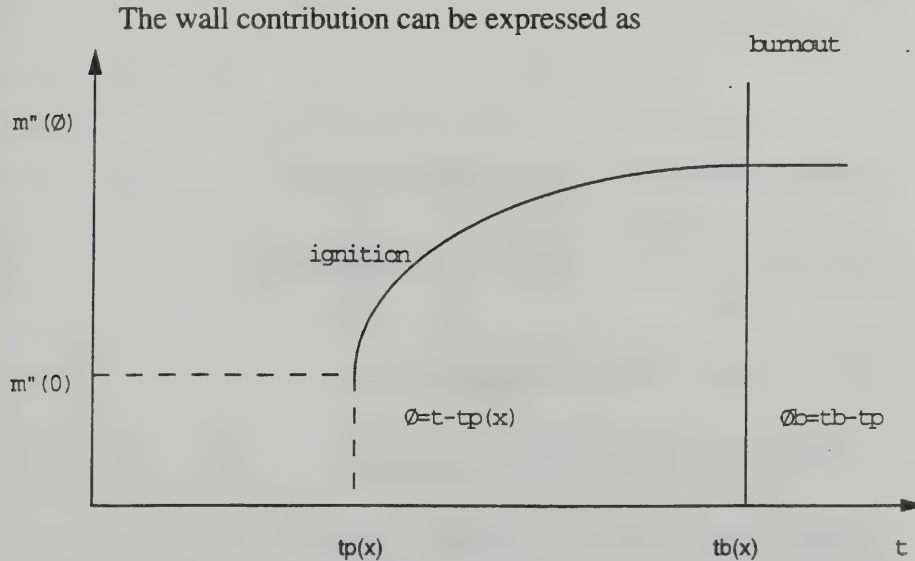
where,  $t_b(x_{fig})$  is the burnout time at  $x=x_{fig}$ , also since  $\dot{Q}_{ig}$  has a fixed duration time( $\Delta t_{ig}$ ),

the ignitor effect can be written as

$$\dot{Q}_{ig} = \eta(t_b(x_{fig}) - t) \cdot \eta(\Delta t_{ig} - t) \cdot \dot{Q}_{ig}. \quad (5.1.6)$$

Eq.(5.1.6) means that the ignitor can affect the flame height before burn out occurs at  $x=x_{fig}$  or before the duration time is achieved.

## 5.2 Representation for the wall contribution ( $\dot{Q}'$ ) and Burning Rate



**Figure 5.2** Illustration of burning rate response to time



$$\dot{Q}' = \Delta H_c \cdot \int_0^{x_p(t)} \dot{m}''(x) dx , \quad (5.2.1)$$

where  $\Delta H_c$  is the heat of combustion of a material (Before we used  $q$  in keeping with Reference[1]).

As shown Figure 5.2 at position  $x$  the wall ignited or began to pyrolyze at time  $t_p(x)$ . This time corresponds to the time when  $x_p(t) = x$ . From previous work[8], we have an implicit formula for  $\dot{m}''(t)$  at  $x$ ,

$$\dot{m}''(\theta)\Delta H_v = \dot{q}_f'' - \sigma T_{ig}^4 - \frac{2k}{\delta}(T_{ig} - T_\infty), \quad (5.2.2)$$

and

$$\theta = t - t_p(x) = \frac{\delta_s^2}{6\alpha} \frac{\Delta H_v}{L} \left[ \frac{\delta_{ig} - \delta}{\delta_s} - \ln \left( \frac{\delta_s - \delta}{\delta_s - \delta_{ig}} \right) \right], \quad (5.2.3)$$

where,

$$\delta_s = \frac{2kL}{c(\dot{q}_f'' - \sigma T_{ig}^4)}, \quad (5.2.4)$$

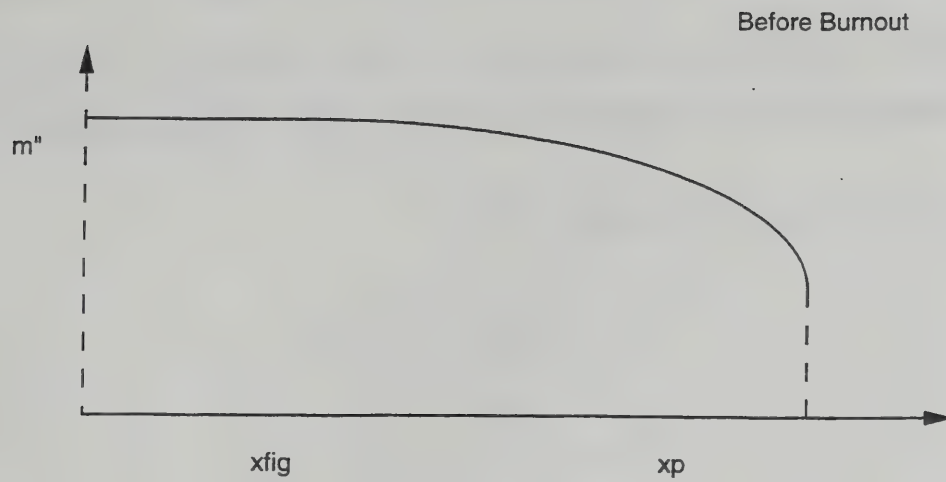
a material constant for a specified flame heat flux,

$$\delta_{ig}(x) = \sqrt{6\alpha(t_p(x) - t_f(x))}, \quad \text{depends on } x \quad (5.2.5)$$

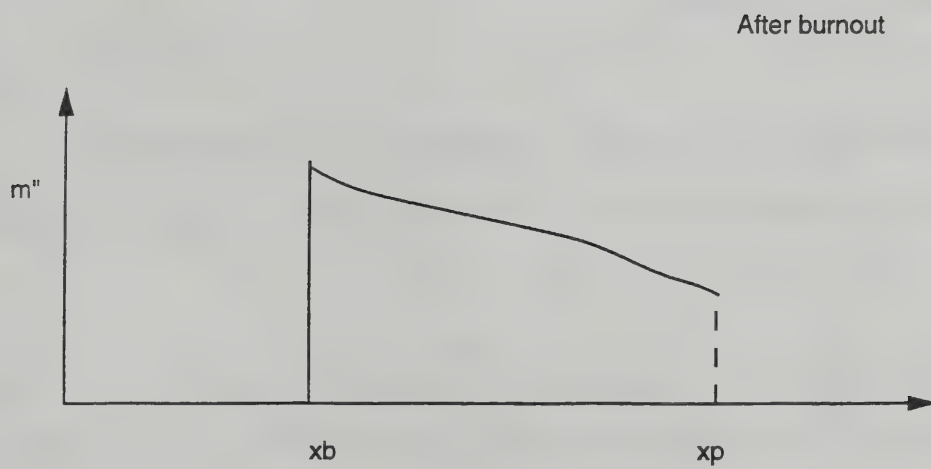
$\Delta H_v$  is the heat of gasfication of a material,

$\delta$  is thermal penetration depth of a material ,

$t_p$  is the time  $x_p(t) = x$  ,  $t_f$  is the time  $x_f(t) = x$ .



(A)



(B)

**Figure 5.3** Burning rate as a function of position

The burning rate model assumes flame heating commences at  $t_f(x)$ , and ignition occurs at  $t_p(x)$ . Each position  $x$  has its own burning history as shown figure 5.3. Note  $t_f(x)$  is the time that  $x$  first experiences a heat flux due to the flame tip reaching  $x$ . The flame spread model assumes a uniform heat flux  $\dot{q}''_f$  from  $x_p$  to  $x_f$  and zero heat flux beyond  $x_f$ , that is  $x > x_f$ . Thus the flame spread model is

$$V_p = \frac{dx_p}{dt} = \frac{x_f - x_p}{\Delta t_f}, \quad (5.2.6)$$

where,  $\Delta t_f = \frac{\pi}{4} \text{ kpc} \left( \frac{T_{ig} - T_\infty}{\dot{q}''_f} \right)^2$ , a flame spread time,

is constant for a given material.

### 5.3 Representation for the wall contribution ( $\dot{Q}'$ ) in terms of $x$

As we discussed in Chapter 3.1, we need to consider the integral equation of Eq.(5.2.1). Let

$$I = \frac{\dot{Q}'}{\Delta H_c} = \int_0^{x_p(t)} \dot{m}''(x,t) dx = \int_0^{x_p(t_{ig})} \dot{m}'' dx + \int_{x_p(t_{ig})}^{x(t)_p} \dot{m}'' dx, \quad (5.3.1)$$

where, the meaning of the first term( $I_1$ ) and second term( $I_2$ ) of R.H.S is the burning rate in ignitor region and above ignitor respectively. Consider each term,

$$I_1 = \int_0^{x_p(t_{ig})} \dot{m}'' dx , \quad (5.3.2)$$

since  $\dot{m}''$  is constant over this region  $0 \leq x \leq x_p(t_{ig}) = x_{fig}$ , Eq.(5.3.2) can be rewritten as

$$I_1 = \dot{m}'' \cdot x_p(t_{ig}) = \dot{m}''(t) \cdot x_{fig} . \quad (5.3.3)$$

And

$$I_2 = \int_{x=x_p(t_{ig})=x_{fig}}^{x=x(t)_p} \dot{m}'' dx , \quad (5.3.4)$$

where, we need to convert to an integral over time. We recognize when  $t = t_{ig}$ ,  $x = x_{fig}$  and when  $t = t_p$ ,  $x = x_p$ . These relationships are the corresponding integral limits.

Because  $\dot{m}''(x,t) = \dot{m}''(\theta)$  , where  $\theta = t - t_p(x)$ , following the same process

from Eq.(3.1.1) to Eq.(3.1.4), the relationship  $V_p = dx_p/dt$  allows

$$I_2 = \int_{t_{ig}}^t \dot{m}''(t - t_p(x)) \cdot V_p(t_p(x)) dt_p . \quad (5.3.5)$$

Since  $\dot{m}''(x,t)$  is the burning rate per unit area at position  $x$  and at time  $t$ , if we know when  $x$  started to pyrolyze  $t_p(x)$ , we can write down this value from our implicit formula,

$$\dot{m}''(x,t) = \dot{m}''(t - t_p(x)) = \dot{m}''(\theta(x)) , \quad (5.3.6)$$

Also, it is convenient to introduce  $\tau$  where,

$$\tau \equiv t - t_{ig} . \quad (5.3.7)$$

Therefore,  $I_2$  becomes

$$I_2 = \int_0^\tau \dot{m}''(\theta) \cdot V_p(\tau_p) d\tau_p , \quad (5.3.8)$$

where,

$$\theta = \frac{\delta_s^2}{6\alpha} \frac{\Delta H_v}{L} \left[ \frac{\delta_{ig} - \delta}{\delta_s} - \ln \left( \frac{\delta_s - \delta}{\delta_s - \delta_{ig}} \right) \right] ,$$

and

$$\dot{m}''(\theta) = \left[ \dot{q}_f'' - \sigma T_{ig}^4 - \frac{2k}{\delta} (T_{ig} - T_\infty) \right] / \Delta H_v .$$

#### 5.4 Burnout Effect

We must limit  $\dot{m}''$  due to burn out. For  $I_1$ ,  $\dot{m}''(t)$  is obtained from the formula  $\dot{m}''(\theta)$  where  $\theta = t - t_p(x_{fig}) = t - t_{ig} = \tau$ . Also burnout occurs after a duration  $\theta_b(x_{fig})$  that is the duration for  $x=x_{fig}$  the initial value. Hence the time for burnout is

$$t_b(x_{fig}) = \theta_b(x_{fig}) + t_{ig} , \quad (5.4.1)$$

or

$$\tau(x_{fig}) = \theta_b(x_{fig}) = t_b(x_{fig}) - t_{ig} . \quad (5.4.2)$$

As long as  $\tau_b(x_{fig})$  is greater than  $\tau$  this region continuous to burn. Hence we write



$$I_1 = \dot{m}''(\tau) \cdot \eta(\tau_b(x_{fig}) - \tau) \cdot x_{fig} , \quad (5.4.3)$$

or

$$I_1 = \dot{m}''(\theta) \cdot \eta(\theta_b(x_{fig}) - \theta(x)) \cdot x_{fig} , \quad (5.4.4)$$

where,  $\theta = \tau$

and  $\theta(x) = \tau - \tau_p(x)$ .

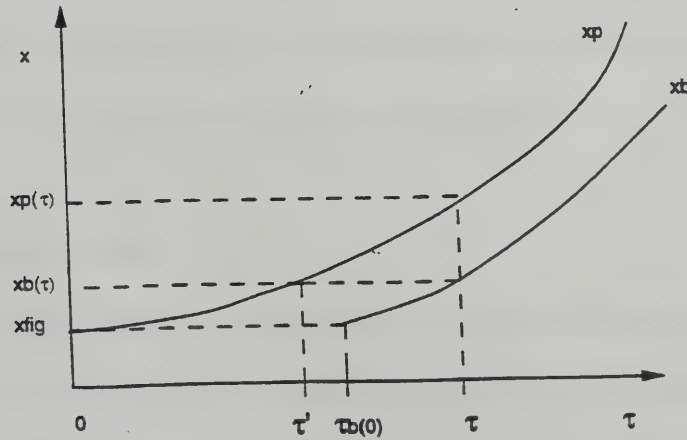
Similarly  $I_2$  becomes

$$I_2 = \int_0^\tau \left[ \eta(\theta_b(x) - \theta) \cdot \dot{m}''(\theta) \right] \cdot V_p(\tau_p) d\tau_p , \quad (5.4.5)$$

where, the burnout time is found by knowing the burning rate at that position  $x_p(\tau)$ ,

$$m'' = \int_0^{\theta_b(x)} \dot{m}''(\theta) d\theta , \quad (5.4.6)$$

and  $m''$  is the burnable mass per unit area ( $g/m^2$ ). This can be found by the density( $\rho$ ) of the wall fuel and its thickness( $\delta$ ) provided all the fuel vaporizes( $m'' = \rho \delta$ ).



**Figure 5.4** The relationship between pyrolysis height and burnout position

The burnout position( $x_b(\tau)$ ) can be found as

$$x_b(\tau) = 0 \quad \text{for } \tau < \tau_b(0), \quad (5.4.7)$$

which is before burnout of region  $0 \leq x \leq x_{\text{fig}}$ , or

$$x_b(\tau) = x_p(\tau') \quad \text{for } \tau \geq \tau_b(0), \quad (5.4.8)$$

which is after burnout of region  $x_{\text{fig}} \leq x \leq x$ .

Figure 5.4 shows the relationship between  $x_p$  and  $x_b$  following Eq.(5.4.7) and (5.4.8),

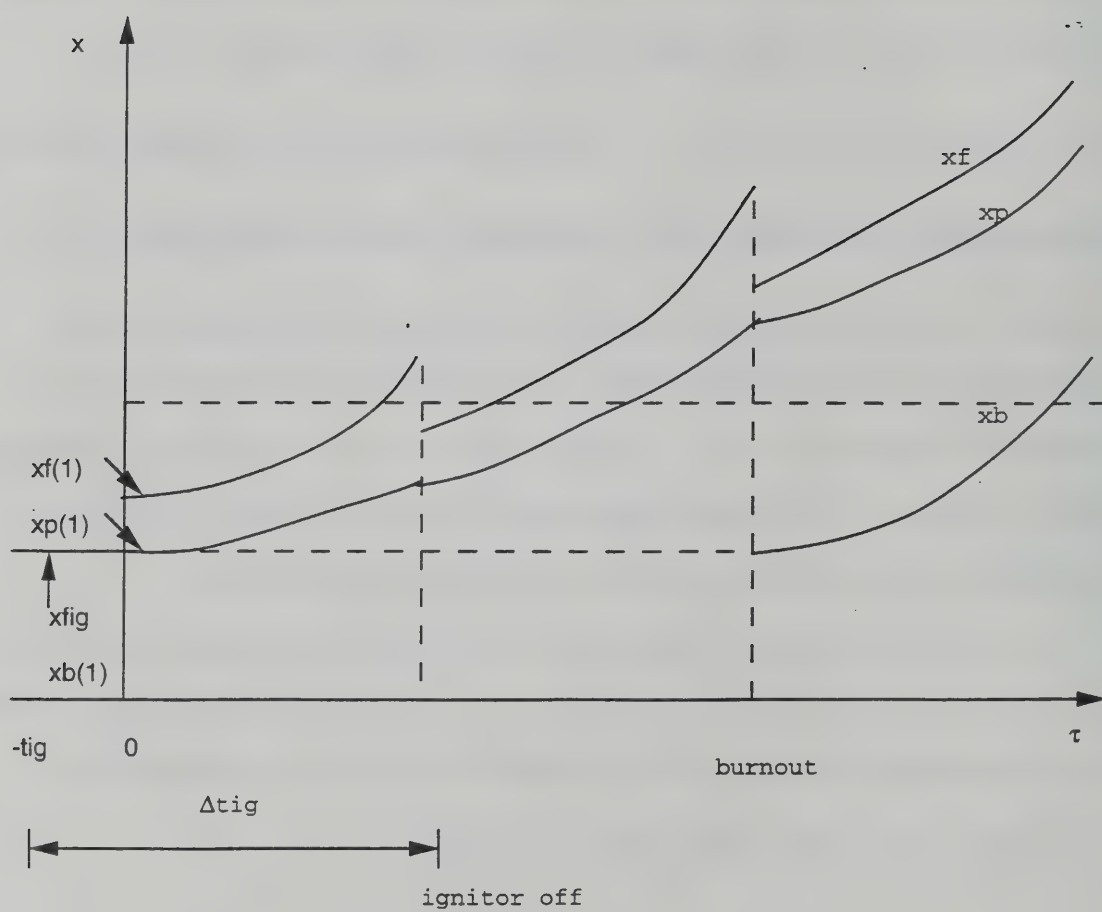
where  $x_b = x_{\text{fig}}$  at  $\tau_b(0)$  and  $\tau_b(x_p(\tau'))$  is burnout time for  $x_p$  at  $\tau'$ .

Reviewing the theory of generalized flame spread model, a computer program can be developed. This program is more complicated than the program discussed in chapter 4 since it includes the ignitor effect, transient burning rate, and burnout effect. From this program, we can obtain the pyrolysis zone( $x_p$ ), the flame height( $x_f$ ), the burnout position( $x_b$ ), the burnout time( $\tau_b$ ), the total energy release rate( $\dot{Q}$ ), and flame velocity( $V_p$ ) of a material at specific time( $\tau$ ). The program can be divided into four parts: (1) Declaration Part, (2) Initial condition, (3) Main Loop, and (4) Subroutines. Also the subroutines are separated into five parts, (1) ROOM to find steady penetration depth, (2) BURNOUT to find burnout time, (3) SEARCHB to find burnout position, (4) SPREAD to find pyrolysis zone, flame height, total energy release rate, and flame velocity, and (5) SEARCHF to find time of arrival of flame tip a material at specific time.

As shown Figure 6.1 we can predict a typical result of generalized flame spread model, where time zero indicates the ignition time and  $x(1)$  are the initial values. The first drop and the second drop of  $x_f$  occur when the ignitor is off and the burnout occurs respectively. The results of this model is in Appendix E

### 6.1 Declaration part

This part includes Input data which has the data of Material, Ignitor Characteristic, Heat Flux, Flame Height, and Computational Parameters, and includes the declaration of variables used for iteration, Computed Parameters, Maximum Number of Steps, and



**Figure 6.1** The typical result of generalized flame spread model

Common Statement. Also this part includes the initialization part which has the data computed using input data of Material, Flame Height, and Computed Parameters.

The material used for this example calculation is PMMA and properties of the material are picked up from Quintiere and Rhodes'[6]. Energy output rate used for the ignitor characteristics, that is the size of the ignitor, and the flame height is picked from Back, et al.[9]. Also we chose the duration of ignition as 200s, that means the ignitor is turned off after 200s, and the width of wall heated as 0.5m. Heat flux from ignitor or wall flame is picked from the results of Williams, et al.[10]. All of data and variables used are shown Appendix C.

## 6.2 Calculation Process

This part is based on the theory of generalized flame spread model, and the program for this spread model will follow this process.

### 6.2.1 Initial Conditions

$$i = 1$$

$$\tau(i) = 0$$

$$\tau_f(i) = -t_{ig}$$

CALL BURNOUT (i,  $\tau(i)$ ,  $\tau_b(i)$ , m") → to find burnout time ( $\tau_b(i)$ ) at initial

position

$$x_b(i) = 0$$

$$x_f(i) = x_b(i) + k_f (\dot{Q}'_{ig} + \dot{Q}'_1) ,$$



where,  $\dot{Q}'_{ig} = \eta (\tau_b(1) - \tau(i)) \cdot \eta (\Delta t_{ig} - t_{ig} - \tau(i)) \dot{Q}'_{ig}$

and  $\dot{Q}'_1 = \Delta H_c \cdot \dot{m}''(i,i)$ ,

CALL ROOTM (i,  $\tau(i)$ , i,  $\tau(i)$ ,  $\tau_f(i)$ ,  $\tau_b(i)$ ,  $\dot{m}''(i,i)$ ) → to find burning rate  $\dot{m}''(i,i)$

$$x_p(i) = x_{fig}$$

$$V_p(i) = \frac{x_f(i) - x_p(i)}{\Delta t_f}$$

$$\dot{Q}(i) = (\dot{Q}'_{ig} + \dot{Q}'_1) w$$

### 6.2.2 Main Loop

We have computed i values and we seek i+1

$$\tau(i+1) = \tau(i) + h$$

Since we need  $\tau_f(i)$  to begin and it depends on knowing  $x_p(i+1)$  we must “guess” by using

the previous value. This only affects the calculation of  $\delta_{ig}(i+1)$  and should not present a

significant error as long as h is small. We will compute  $\tau_f(i+1)$  after finding  $x_p(i+1)$ .

$$\tau_f(i+1) = 0, \quad x_p(i) \leq x_f(i), \text{ the flame is heating } x_{fig} < x \leq x_f(1) \text{ from time } 0$$

or

$$\tau_f(i+1) = \tau_f(i)$$

CALL BURNOUT (i+1,  $\tau(i+1)$ ,  $\tau_f(i+1)$ ,  $\tau_b(i+1)$ , m") → to find burnout time

( $\tau_b(i+1)$ ) at time i+1

$x_b(i+1) = 0$  ,  $\tau(i+1) < \tau_b(1)$  , region  $0 \leq x \leq x_{fig}$  has not burned out

or

$x_b(i+1) = x_p(i)$  ,  $\tau(i+1) \geq \tau_b(1)$

CALL SEARCHB (  $\tau(i+1)$ ,  $x_b(i+1)$ )

This finds  $x_b(i+1) = x_p(i)$  where  $\tau_b(j) = \tau(i+1)$ .

CALL SPREAD (i+1,  $V_p(i+1)$ ,  $x_f(i+1)$ ,  $x_p(i+1)$ ,  $\dot{Q}(i+1)$ )

CALL SEARCHF (i+1,  $x_p(i+1)$ ,  $\tau_f(i+1)$ )

### 6.2.3 Subroutine ROOTM (i, $\tau(i)$ , j, $\tau_p(j)$ , $\tau_f(j)$ , $\tau_b(j)$ , $\dot{m}''(i,j)$ )

This finds burning rate  $\dot{m}''(i,j)$ .  $\tau(i)$  is the current time and  $\tau_p(j)$  is the time

corresponding to  $x_p(\tau)=x$ . From Eq.(5.2.2), Eq.(5.2.3), and Eq.(5.2.5),

$$\dot{m}''(i,j) = \frac{\eta(\tau_b(j) - (i))}{\Delta H_v} \left[ \dot{q}'' - \frac{2k}{\delta} (T_{ig} - T_\infty) \right] ,$$

$$\delta = \delta_s - (\delta_s - \delta_{ig}) \exp \left[ \left( \frac{\delta_{ig} - \delta}{\delta_s} \right) - \left( \frac{\tau(i) - \tau_p(j)}{\tau^*} \right) \right] ,$$

Where,  $\dot{q}'' = \dot{q}''_f - \sigma T_{ig}^4$  , net flame heat flux,

$$\delta_s = \frac{2kL}{c(\dot{q}''_f - \sigma T_{ig}^4)} , \text{ a material constant for a specified flame heat flux,}$$

and  $\tau^* = \frac{\delta_s^2}{6\alpha} \frac{\Delta H_v}{L}$  , burn time constant,

and

$$\delta_{ig}(x) = \sqrt{6\alpha(t_p(x) - t_f(x))} .$$

To find  $\delta$  the program will use an iteration loop. For first guess to iterate, a previous value is used.

#### 6.2.4 Subroutine BURNOUT (j, $\tau_p(j)$ , $\tau_f(j)$ , $\tau_b(j)$ , m'')

This finds burnout time ( $\tau_b$ ).  $\tau_f(i)$  is the time of arrival of flame tip at position  $x=x_p(i)$  and m'' is a burnable mass per unit area. From Eq.(5.4.6),

$$m'' = \int_0^{\tau_b(i)} \dot{m}''(i,j) d\tau .$$

Using Trapezoidal Rule to solve the integral, this equation can be rewritten as

$$m'' = \frac{h}{2} \sum_{i=1}^{n-1} [\dot{m}''(i,j) + \dot{m}''(i+1, j)] .$$

The burnout time is when the integral  $\geq m$  where the last  $i=n-1$  and  $n=i+1$ . That is,  $\tau(n) = (n-1)h$ . Therefore,  $\tau_b(j) = (n-1)h$ . Here, we need subroutine ROOTM to find burning rate  $\dot{m}''(i,j)$ . That is,

ROOTM ( $i, \tau(i), j, \tau_p(j), \tau_f(j), \infty, \dot{m}''(i,j)$ )

ROOTM ( $i+1, \tau(i+1), j, \tau_p(j), \tau_f(j), \infty, \dot{m}''(i+1,j)$ ) ,

where  $\infty$  is the value of burnout time in subroutine ROOTM. Since we are integrating up to the burnout time, we can put this value( $\infty$ ) as a big number.

#### 6.2.5 Subroutine SEARCHB ( $\tau(i), x_b(i)$ )

This seeks  $j$  such that  $\tau_b(j) = \tau(i)$ .

Do  $j = 1, i$

IF [ $\tau_b(j) < \tau(i)$ ] Then Continue Do Loop

Else  $x_b(i) = x_p(j)$  , for  $\tau_b(j) \geq \tau(i)$

Return

End

#### 6.2.6 Subroutine SPREAD ( $j, V_p(j), x_f(j), x_p(j), \dot{Q}(j)$ )

This find the velocity of the pyrolysis front,  $V_p(j)$ , the flame tip position,  $x_f(j)$ , the pyrolysis front position,  $x_p(j)$ , and the total energy release rate,  $\dot{Q}(j)$ .

$$\dot{Q}'_{ig} = \eta (\tau_b(1) - \tau(i)) \cdot \eta (\Delta t_{ig} - t_{ig} - \tau(i)) \dot{Q}'_{ig}$$

$$\dot{Q}'_1 = \Delta H_c \cdot \dot{m}''(j,1) \cdot x_{fig}.$$

We need initial guess like  $V_p(j) = V_p(j-1)$  to find a real  $V_p(j)$  by an iteration loop.

$$FI(j) = 2/h [ FI(j-1) + \dot{m}''(j, j-1) V_p(j-1) + \dot{m}''(j,j) V_p(j)] ,$$

$$\text{where,} \quad FI(j-1) = \frac{h}{2} \sum_{k=1}^{j-2} (\dot{m}''(j,k) V_p(k) + \dot{m}''(j,k+1) V_p(k+1)) ,$$

is can be gotten following the same step described in Chapter 3.2. We need call ROOTM to find burning rate at that time and position.

$$\dot{Q}'_2 = \Delta H_c FI(j)$$

$$x_f(j) = k_f (\dot{Q}'_{ig} + \dot{Q}'_1 + \dot{Q}'_2)^n + x_b(j) .$$

Following the same way discussed in Chapter 3.2 to solve integral for  $x_p$ ,

$$x_p(j) = C + (h/2) V_p(j) ,$$

$$\text{where,} \quad C = x_{fig} + \frac{h}{2} \left\{ \sum_{k=1}^{j-2} (V_p(k) + V_p(k+1)) + V_p(j-1) \right\} .$$

$$V_p(j) = \frac{x_f(j) - x_p(j)}{\Delta t_f} .$$

$$\text{Error} = \frac{|V_p(j) - V_p(j-1)|}{V_p(j)} \leq \epsilon .$$



When this condition is satisfied, we can get a new correct value( $V_p(j)$ ).

#### 6.2.7 Subroutine SEARCHF ( $j, x_p(j), \tau_f(j)$ )

This finds the time when the flame tip first reached  $x=x_p(j)$ .

Do  $k = 1, j$

IF [ $x_f(k) < x_p(j)$ ] Then Continue Do Loop

Else  $x_f(k) = x_p(j)$

$\tau_f(j) = (k-1) h$

Return

End

#### 6.2.8 The Program of Generalized Spread Model

The program follows the process described above. To make the program simple we use common statements, and the time step( $H$ ) is 1.0 second.

Subroutine ROOTM is called by subroutine BURNOUT and SPREAD to find burning rate as described above. However, burnout time,  $\tau_b$ , has a different value when

ROOTM is called by these subroutines. For example, once ROOTM is called from

BURNOUT, we put  $\tau_b$  with an “infinity”(big) value, however,  $\tau_b$  is put with its true

value, that is found in BURNOUT, when ROOTM is called from SPREAD. Therefore,

the subroutine ROOTM is only used to find  $\delta$ , and whenever the subroutine ROOTM is

required to find burning rate, this program writes down the equation of burning rate after the Call ROOTM statement.

The Program of Generalized Spread Model is shown in Appendix D.

The velocity of flame spread is related to the pyrolysis front position of material,  $x_p(i)$ , the flame tip position,  $x_f(i)$ , and the characteristic ignition time,  $\tau$ , that is affected by  $k\rho c$ , as shown Eq.(2.2.1) and Eq.(2.2.2). In this section we compare the relationship between  $x_p$  and  $x_f$  and the relationship between  $V_p$  and  $x_p$  of the exact solution for  $n=1$ ,  $n=2/3$ , and the generalized flame spread model with the results that others found for PMMA.

### 7.1 The Properties Used for Comparison

Orloff, de Ris, and Markstein [11] reported, in their experimental study,  $k = 0.64 \times 10^{-3} \text{ cal/cm}^2\text{C}^\circ$ ,  $\rho = 1.19 \text{ g/cm}^3$ , and  $c = 0.50 \text{ cal/g}^\circ\text{C}$ , respectively. Therefore  $k\rho c$  is  $0.654 \text{ kW}^2\text{s/m}^4\text{C}^2$ . These values were assumed constant over the temperature range from ambient ( $20^\circ\text{C}$ ) to ignition ( $363^\circ\text{C}$ ) and under heat flux  $25 \text{ kW/m}^2$ . They measured the burning rate of PMMA during upward flame spread finding it varied from 7.2 to 12.0  $\text{g/cm}^2\text{s}$ . Their initial condition was taken as 0.02 m. We chose an average burning rate of  $9.6 \text{ g/m}^2\text{s}$  and  $x_{po} = 0.02 \text{ m}$  for our “exact constant burning rate solution” comparisons. The variables and data used for the comparison are in Table F.1.

Mitler and Steckler[12] used the LIFT value derived for  $k\rho c$  of PMMA ( $1.02 \text{ kW}^2\text{s/m}^4\text{C}^2$ ) in their study. We will only use this value for this comparison, and the other properties are same with Orloff, et al data. The variables and data used for the comparison

are in Table F.2.

In case of the generalized flame spread model, we use the  $k\rho c$  from the LIFT data and 1kW of energy output rate( $\dot{Q}_{ig}$ ) which is calculated to make  $x_{po}=0.02$  m. The other properties are same with the data described in Appendix C. Using the different ignition temperature( $T_{ig}$ ), also, we try to find the effect of ignition temperature( $T_{ig}$ ) on flame spread. The ignition temperatures( $T_{ig}$ ) used for this are 180°C and 363°C that come from J. Quintiere and B. Rhodes'[6] and L. Orloff, et al.[11] respectively.

Table 7.1 shows the different  $k\rho c$  values used for the comparison.

**Table 7.1**

The  $k\rho c$  properties of PMMA used for the comparison

Source	PROPERTIES			
	k	$\rho$	c	$k\rho c$
Orloff et al	$2.63 \times 10^{-4}$	1190	2.09	0.654
LIFT	$0.346 \times 10^{-3}$	1180	2.5	1.02
Generalized Flame Spread Model	$0.346 \times 10^{-3}$	1180	2.5	1.02
UNITS	kW/m. $^{\circ}$ C	kg/m <sup>3</sup>	kJ/kg. $^{\circ}$ C	kW <sup>2</sup> s/m <sup>4</sup> C <sup>2</sup>

## 7.2 The Relationship between $x_p$ and $x_f$

Orloff, de Ris, and Markstein measured the relationship, labelled experiment result between  $x_p$  and  $x_f$  finding a best fit of

$$x_f = 1.95 x_p^{0.781}, \quad (7.2.1)$$

as shown in Figure 7.1. Using their properties ( $q=25 \text{ kW/m}^2$ ,  $\dot{m}''=9.6 \text{ g/m}^2.\text{s}$ ) and

Eq.(2.2.5) we can find the relationship, labelled exact solution as

$$x_f = 2.4 x_p, \quad \text{for } n=1, \quad (7.2.2)$$

and

$$x_f = 2.59 x_p^{0.667}, \quad \text{for } n=2/3. \quad (7.2.3)$$

Similarly we can also find the relationship using the PMMA LIFT data. The flame height relationships, however, are same with those of Orloff, de Ris, and Markstein because the same  $q$  and  $\dot{m}''$  are used. The results of the flame velocity for these two data, however, will be different since they have the different value of  $k_p c$ . These results will be shown later.

Delichatsios, Mathews, and Delichatsios[13] found the relationship using 0.052 as a flame height coefficient,  $k_p$

$$x_f = 2.01 x_p^{0.667}, \quad (7.2.4)$$

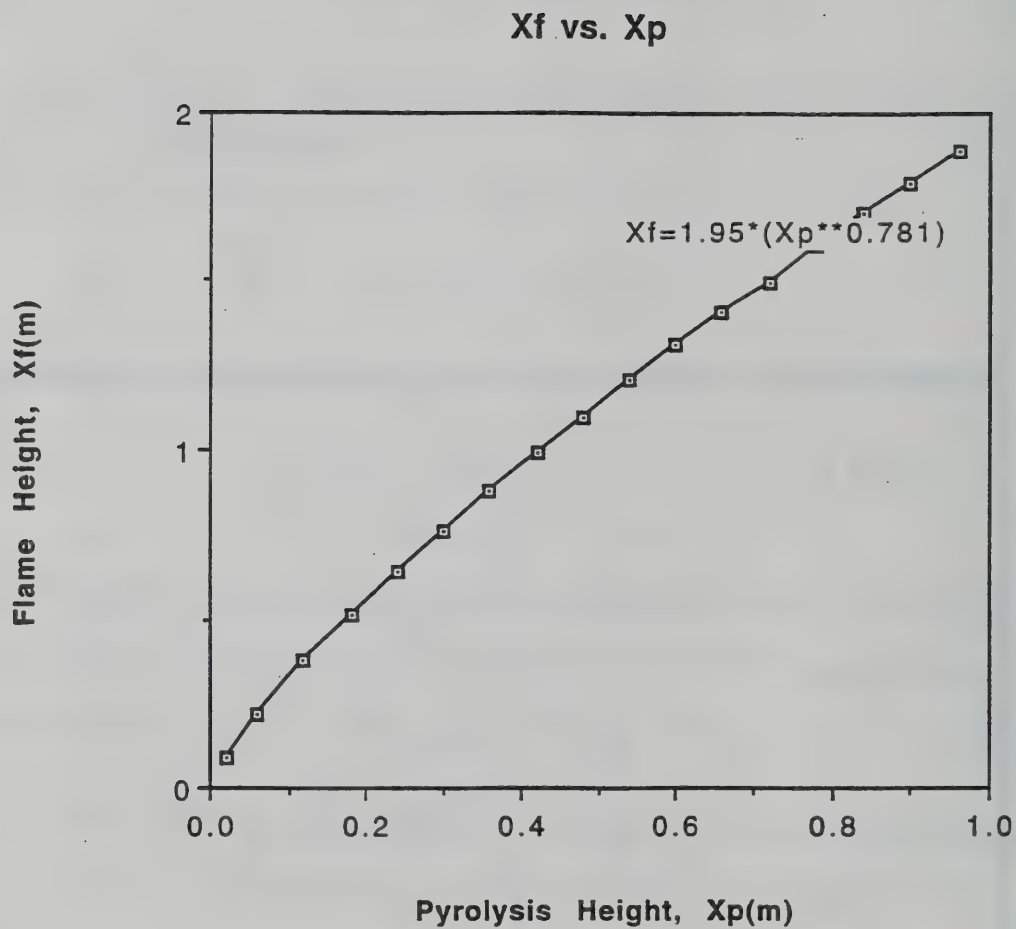
as shown Figure 7.2.

The properties used for the relationships are in Table 7.2

**Table 7.2**

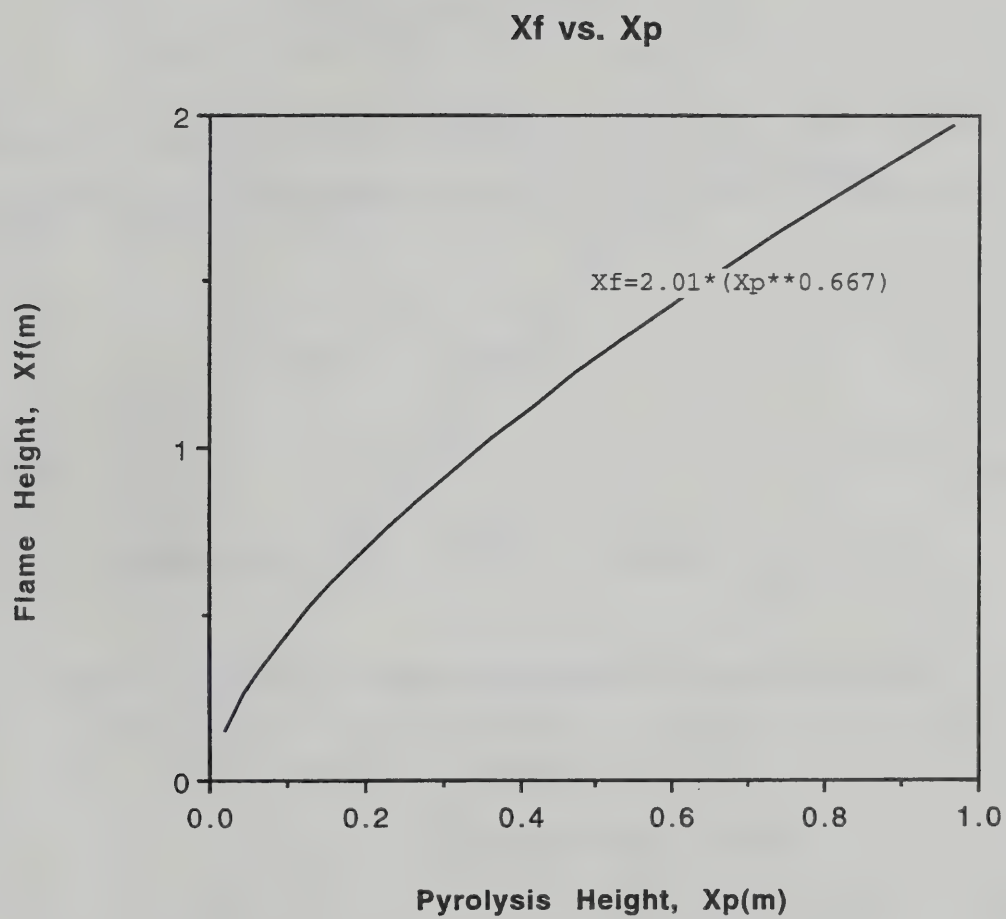
The properties used for the relationship between  
pyrolysis height,  $x_p$ , and flame height,  $x_f$

Source	PROPERTIES				
	n	$k_f$	$\dot{Q}'$	q	$\dot{m}''$
Orloff et al	1	0.01	0	25	9.6
	2/3	0.067	0	25	9.6
LIFT	1	0.01	0	25	9.6
	2/3	0.067	0	25	9.6
Delichatsios et al	2/3	0.052	0	25	9.6
UNITS		$m^2/kW$ $(m^5/kW^2)^{1/3}$	$kW/m$	$kW/m^2$	$g/m^2.s$



**Figure 7.1** The relationship between flame height and pyrolysis height for PMMA by Orloff, de Ris, and Markstein





**Figure 7.2** The relationship between flame height and pyrolysis height for PMMA by Delichatsios, Mathews, and Delichatsios

### 7.3 The Relationship between $V_p$ and $x_p$

Orloff, de Ris, and Markstein measured the relationship called experiment result between  $V_p$  and  $x_p$  with

$$V_p = 0.00441 x_p^{0.964}, \quad (7.3.1)$$

as shown Figure 7.3. Using the relationship of Eq.(7.2.2) and Eq.(7.2.3) and substituting these equation into Eq.(2.2.2) we can find the relationship of  $V_p$  and  $x_p$  in the exact solution as

$$V_p = 0.01448 x_p, \quad \text{for } n=1, \quad (7.3.2)$$

and

$$V_p = 0.0103(2.59 x_p^{0.667} - x_p), \quad \text{for } n=2/3. \quad (7.3.3)$$

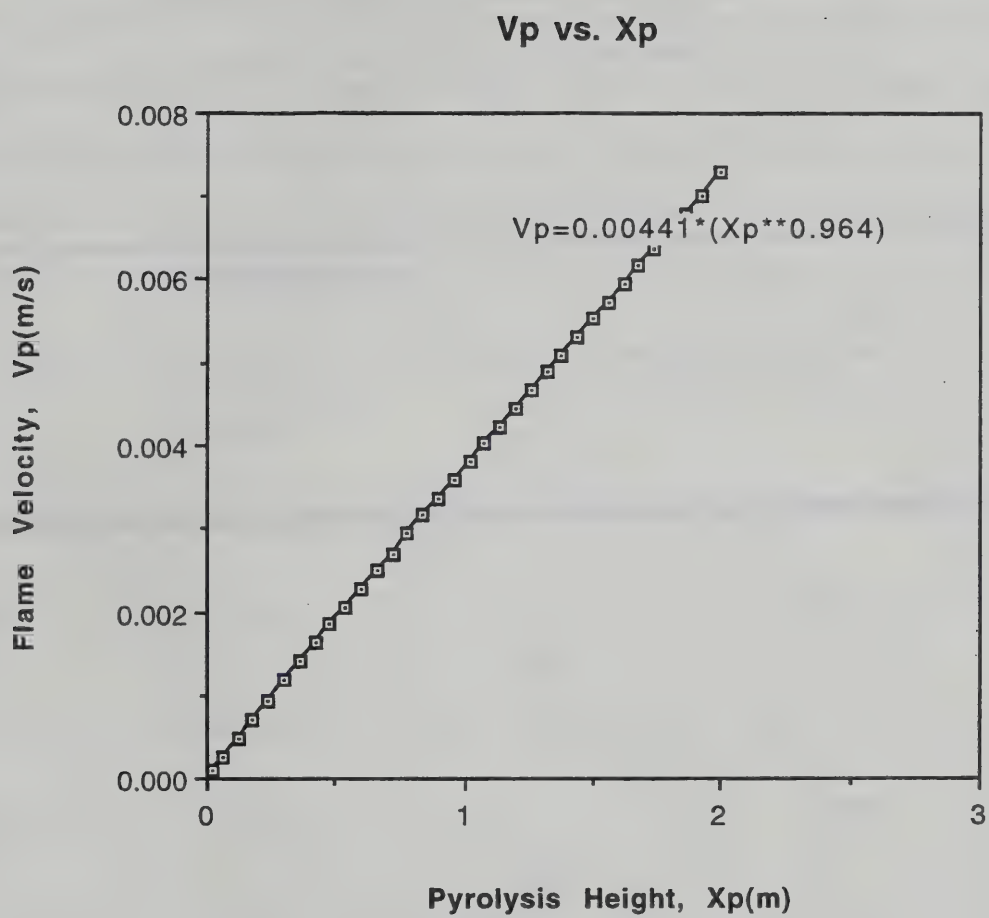
Similarly we can also find the relationship of  $V_p$  and  $x_p$  for LIFT data as

$$V_p = 0.009283 x_p, \quad \text{for } n=1, \quad (7.3.4)$$

and

$$V_p = 0.00663(2.59 x_p^{0.667} - x_p), \quad \text{for } n=2/3. \quad (7.3.5)$$

The properties used for the relationships are in Table 7.3, where  $\tau$  is ignition time.



**Figure 7.3** The relationship between flame velocity and pyrolysis height for PMMA by Orloff, de Ris, and Markstein

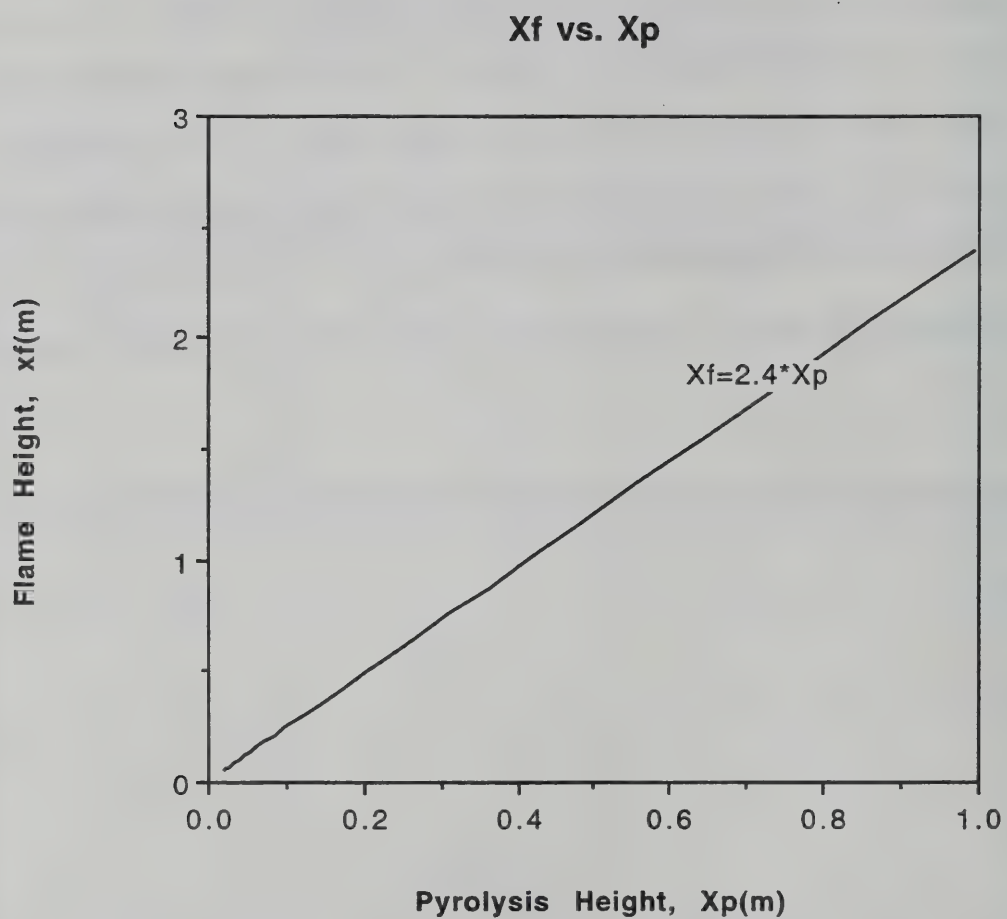
**Table 7.3**

The properties used for the relationship between  
flame velocity,  $V_p$ , and pyrolysis height,  $x_p$ .

Source	PROPERTIES		
	n	k p c	$\tau$
Orloff et al	1	0.654	96.7
	2/3		
LIFT	1	1.02	150.799
	2/3		
UNITS		$\text{kw}^2/\text{s/m}^4\text{C}^2$	sec

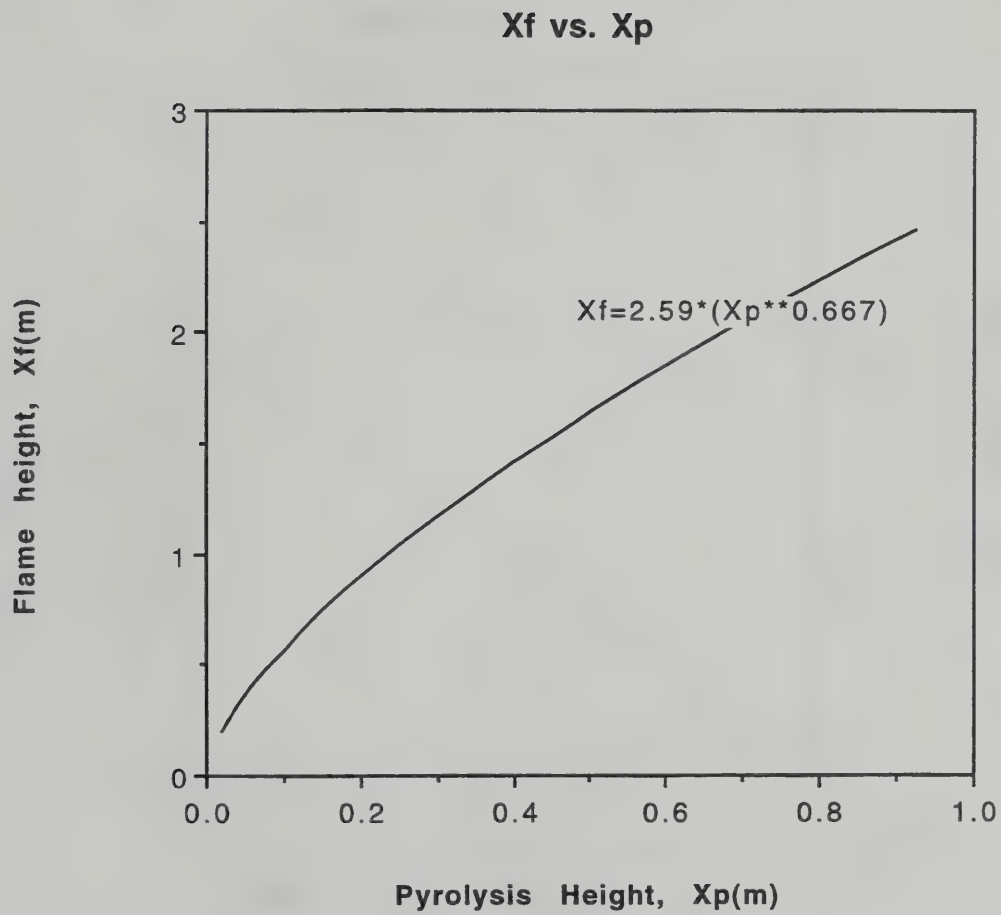
#### 7.4 The Programs used for Comparisons and Results

The programs used for comparison are in Appendix G. The relationship of pyrolysis height,  $x_p$ , and flame height,  $x_f$  and flame velocity,  $V_p$ , and pyrolysis height,  $x_p$ , are shown Figure 7.4 - 7.13. Figure 7.8 is the result of the comparison of flame height and pyrolysis height between the exact solutions and the experiment and the generalized flame spread model. These curves in figure 7.8 show the effect of the different flame height coefficient and power to the flame height. Figure 7.13 is the result of the comparison of flame velocity and pyrolysis height between the exact solutions and the experiment and the generalized flame spread model. These curves in figure 7.13 also show the effect of the different  $k_p c$  and ignition temperature( $T_{ig}$ ) to the flame velocity.

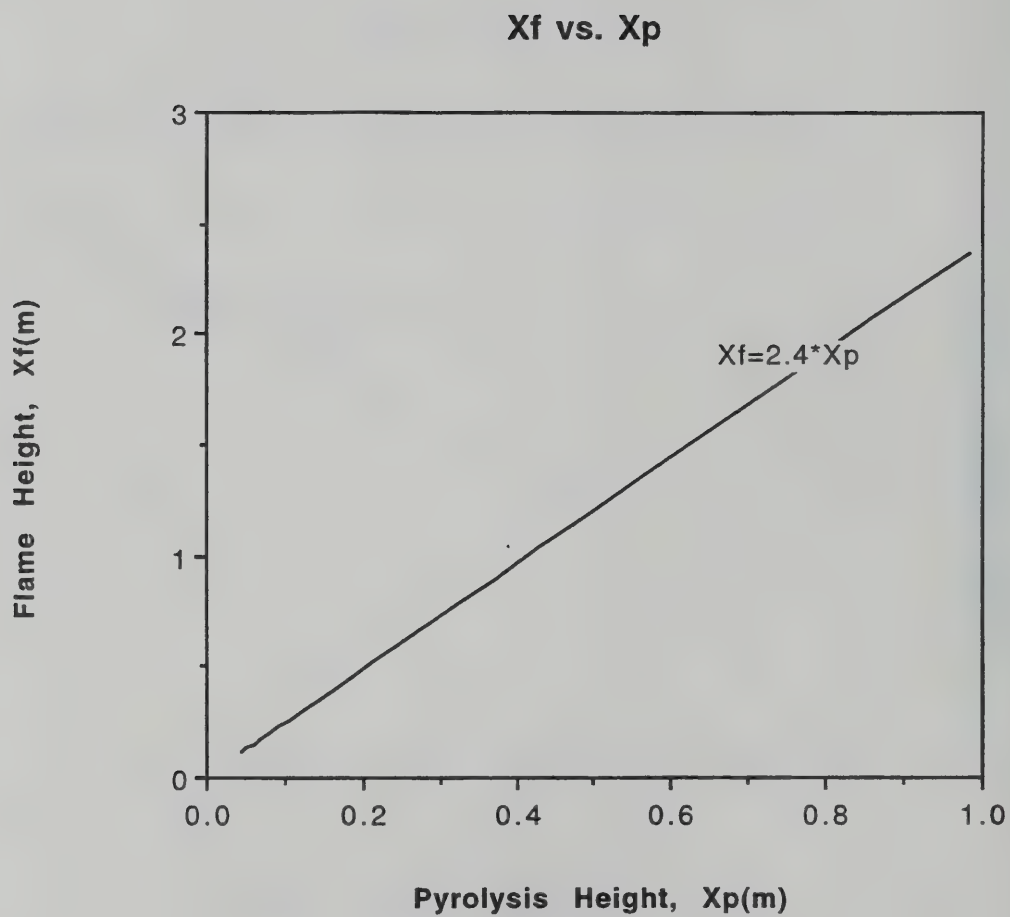


**FIGURE 7.4** The result of flame height vs. pyrolysis height used exact solution for  $n=1$  with Orloff, de Ris, and Markstein data.

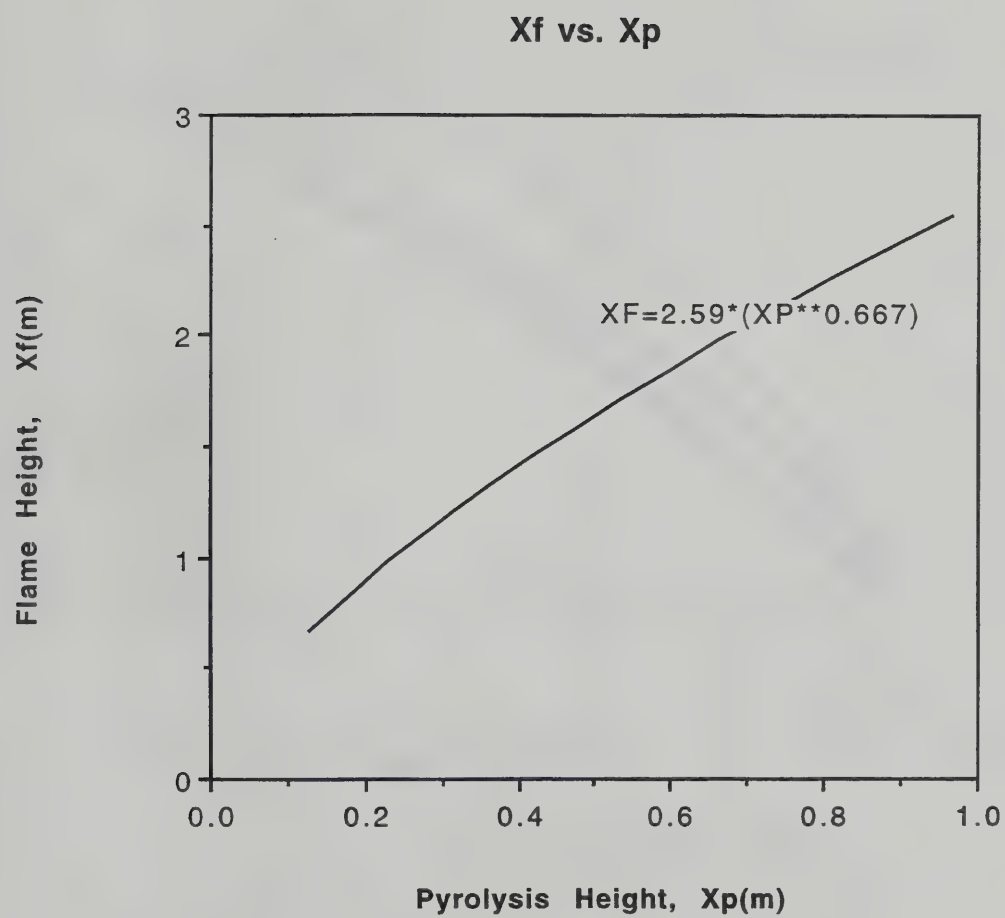




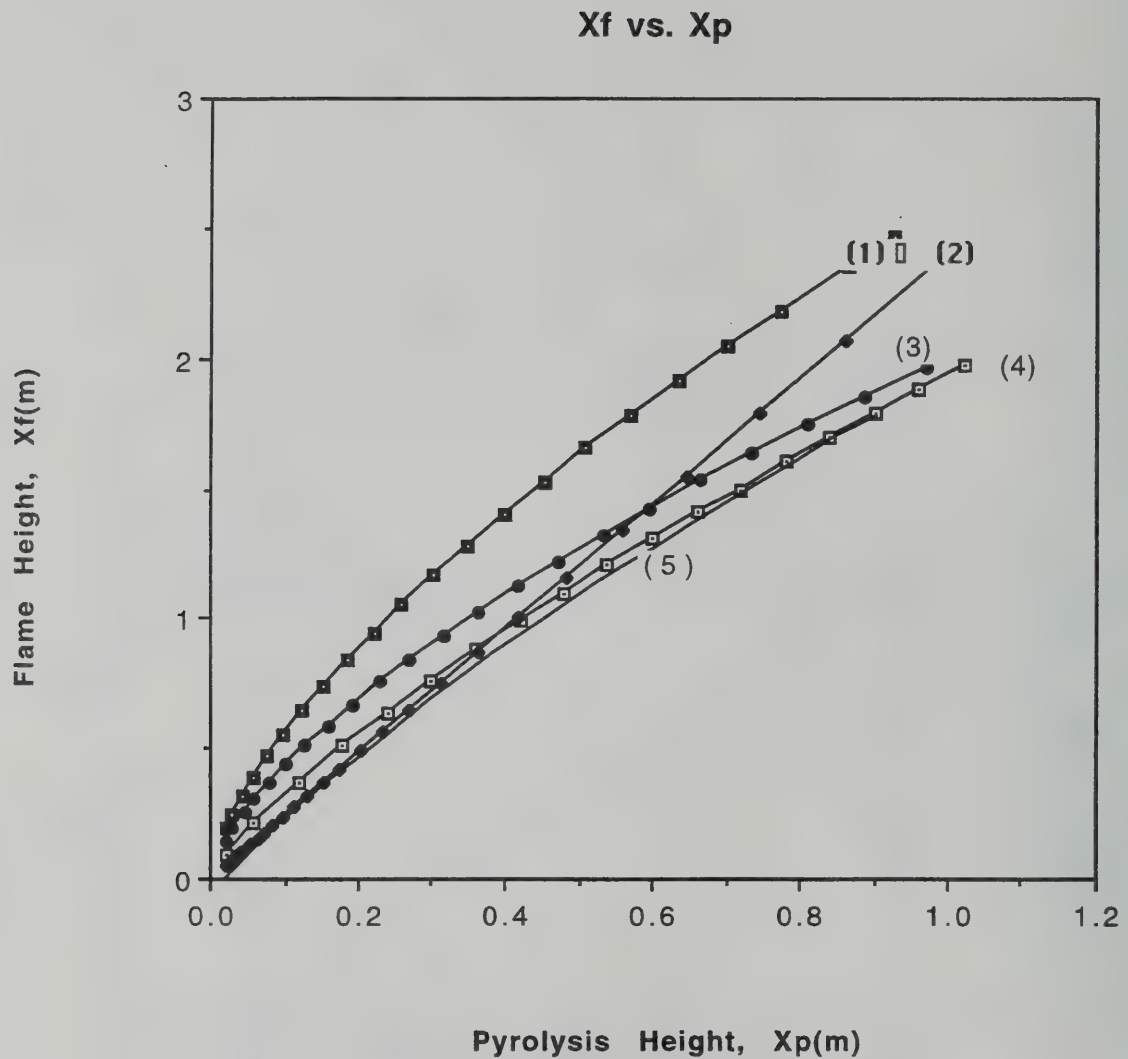
**FIGURE 7.5** The result of flame height vs. pyrolysis height used exact solution for  $n=2/3$  with Orloff, de Ris, and Markstein data.



**FIGURE 7.6** The result of flame height vs. pyrolysis height used exact solution for  $n=1$  with LIFT data.

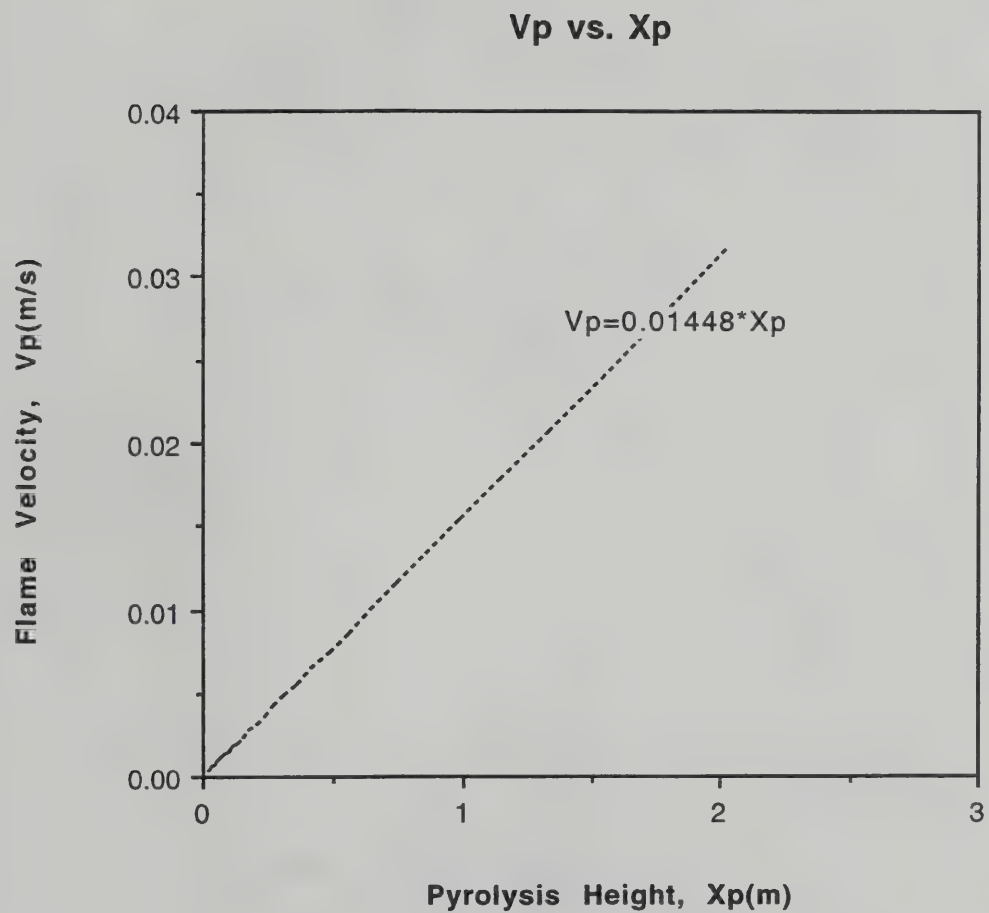


**FIGURE 7.7** The result of flame height vs. pyrolysis height used exact solution for  $n=2/3$  with LIFT data.

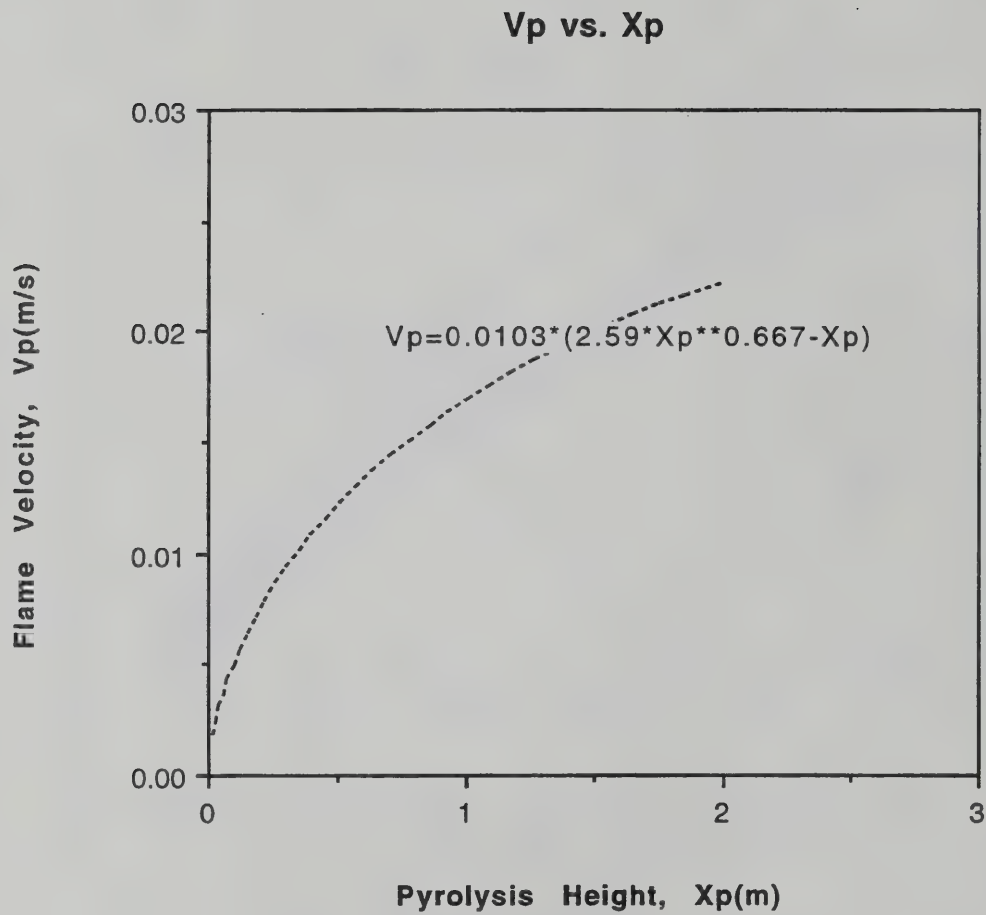


- (1)  $x_f = 2.59 * (x_p^{**0.667})$  , Exact solution for  $n=2/3$ , based on constant  $\dot{m}''$
- (2)  $x_f = 2.4 * x_p$ , Exact solution for  $n=1$ , based on constant  $\dot{m}''$
- (3)  $x_f = 2.01 * (x_p^{**0.667})$ , Delichatsios, Mathews, and Delichatsios, based on constant  $\dot{m}''$
- (4)  $x_f = 1.95 * (x_p^{**0.781})$ , Experiment by Orloff, de Ris, and Markstein
- (5) Generalized flame spread model based on transient  $\dot{m}''$

**FIGURE 7.8** The comparison of flame height vs. pyrolysis height.

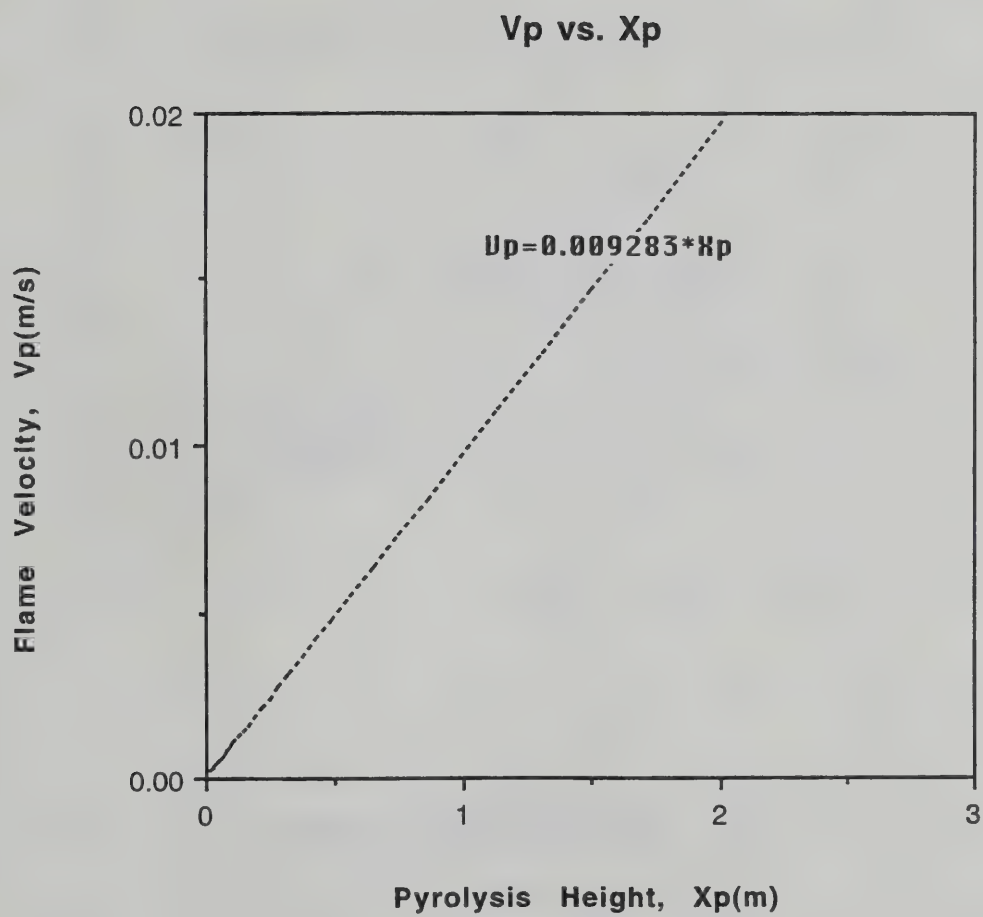


**FIGURE 7.9** The result of flame velocity vs.flame height used exact solution for  $n=1$  with with Orloff, de Ris, and Markstein data.

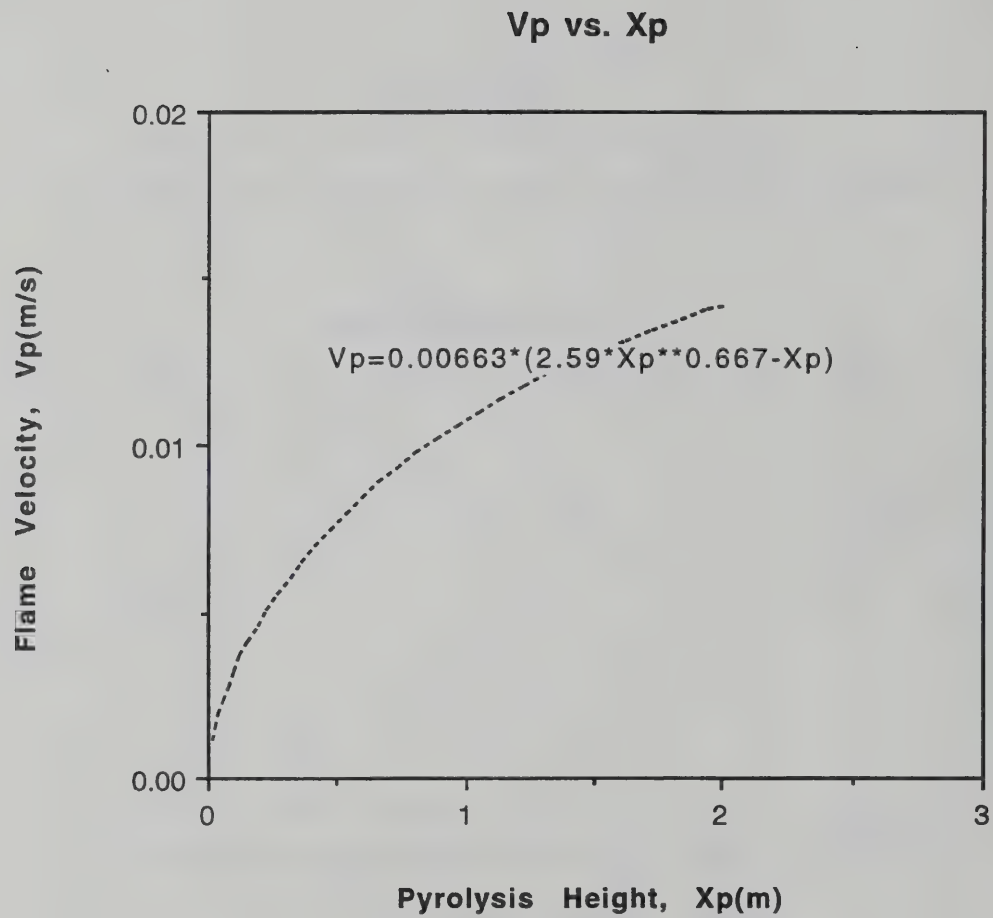


**FIGURE 7.10** The result of flame velocity vs.flame height used exact solution for  $n=2/3$  with Orloff, de Ris, and Markstein data.



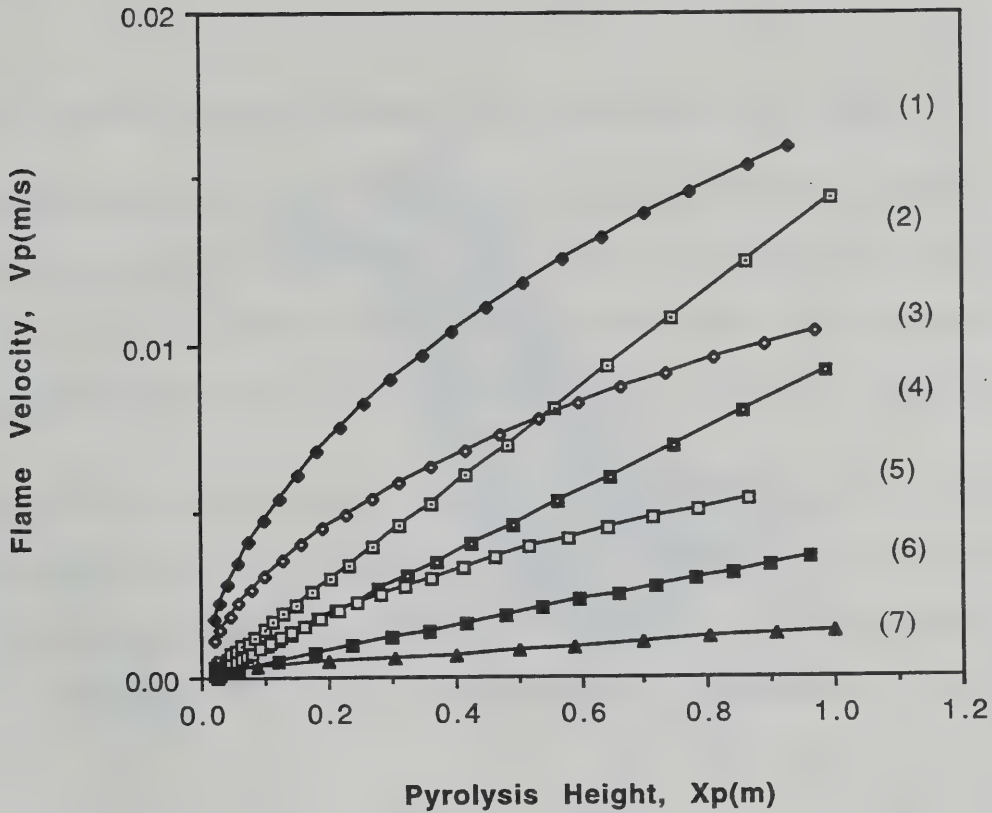


**FIGURE 7.11** The result of flame velocity vs.flame height used exact solution for  $n=1$  with LIFT data.



**FIGURE 7.12** The result of flame velocity vs. flame height used exact solution for  $n=2/3$  with LIFT data.

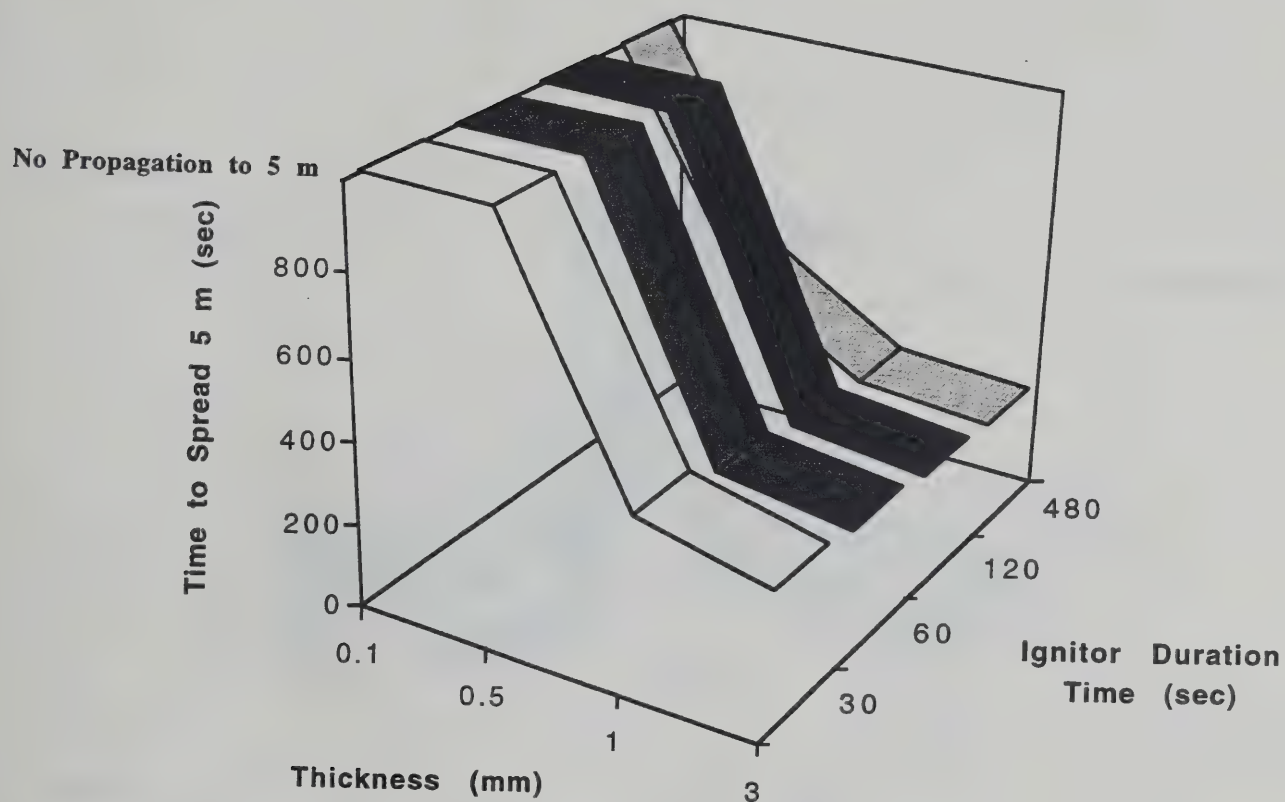
## V<sub>p</sub> vs. X<sub>p</sub>



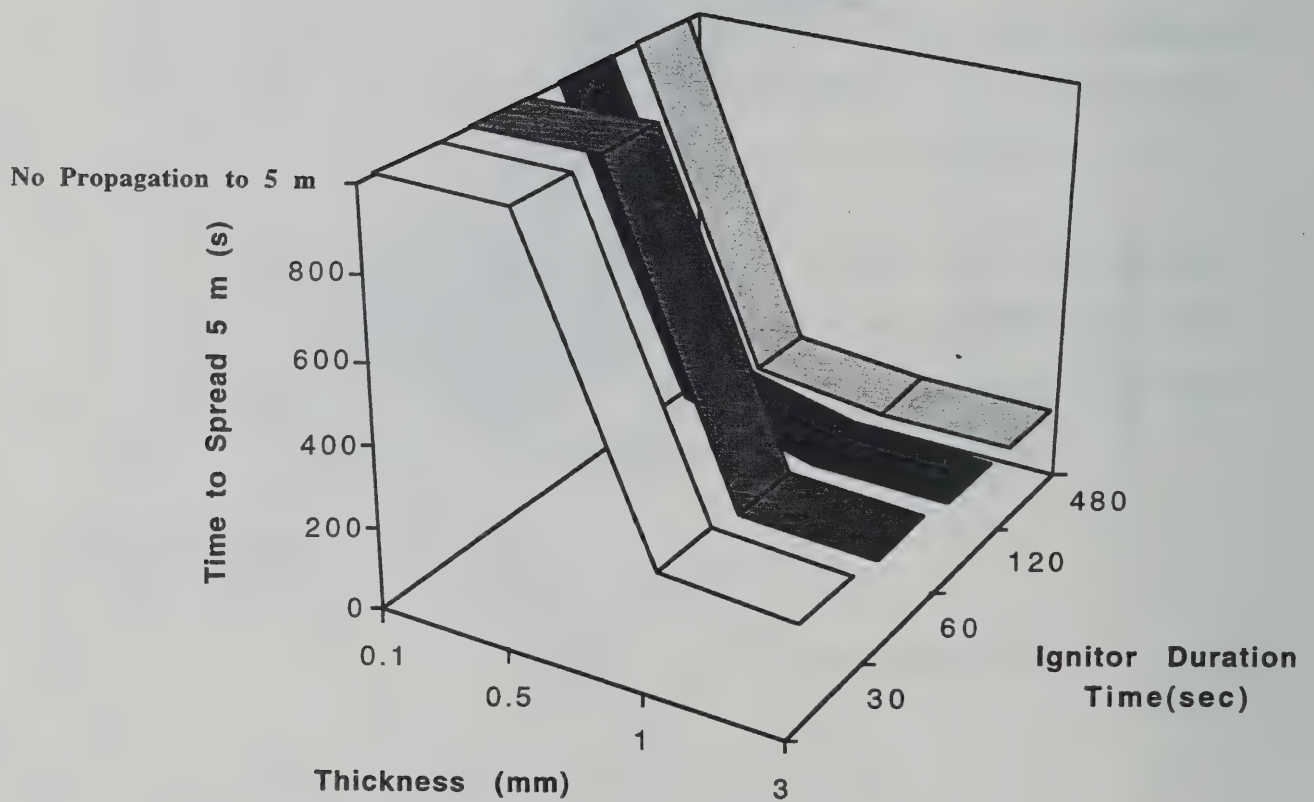
- (1)  $V_p = 0.0103 \cdot (2.59x_p^{0.667} - x_p)$ , Exact solution for  $n=2/3$  with Orloff, de Ris, and Markstein data,  $k_p c = 0.654 \text{ kW}^2\text{s/m}^4\text{C}^2$ ,  $\dot{m}'' = \text{const}$ ,  $T_{ig}=363^\circ\text{C}$
- (2)  $V_p = 0.01448 \cdot x_p$ , Exact solution for  $n=1$  with Orloff, de Ris, and Markstein data,  $k_p c = 0.654 \text{ kW}^2\text{s/m}^4\text{C}^2$ ,  $\dot{m}'' = \text{const}$ ,  $T_{ig}=363^\circ\text{C}$
- (3)  $V_p = 0.00663 \cdot (2.59x_p^{0.667} - x_p)$ , Exact solution for  $n=2/3$  with LIFT data,  $k_p c = 1.02 \text{ kW}^2\text{s/m}^4\text{C}^2$ ,  $\dot{m}'' = \text{const}$ ,  $T_{ig}=363^\circ\text{C}$
- (4)  $V_p = 0.009283 \cdot x_p$ , Exact solution for  $n=1$  with LIFT data,  $k_p c = 1.02 \text{ kW}^2\text{s/m}^4\text{C}^2$ ,  $\dot{m}'' = \text{const}$ ,  $T_{ig}=363^\circ\text{C}$
- (5) Generalized flame spread model,  $k_p c = 1.02 \text{ kW}^2\text{s/m}^4\text{C}^2$ , transient  $\dot{m}''$ ,  $T_{ig}=180^\circ\text{C}$
- (6)  $V_p = 0.00441 \cdot (x_p^{0.964})$ , Experiment by with Orloff, de Ris, and Markstein  $7.2\text{g/m}^2.\text{s} \leq \dot{m}'' \leq 12\text{g/m}^2.\text{s}$ ,  $T_{ig}=363^\circ\text{C}$
- (7) Generalized flame spread model,  $k_p c = 1.02 \text{ kW}^2\text{s/m}^4\text{C}^2$ , transient  $\dot{m}''$ ,  $T_{ig}=363^\circ\text{C}$

**FIGURE 7.13** The comparison of flame velocity vs.flame height.

Using the generalized flame spread model with  $k\rho c=1.02 \text{ kW}^2\text{s/m}^4\text{C}^2$  and the properties described by Quintiere and Rhodes [6] in Appendix C, we try to find the effect of thickness and the ignitor on flame spread in this section. A study on the effect of thickness and the ignitor include variations of thickness(mm): 0.1, 0.5, 1.0, 3.0 ; ignitor duration(s) : 30, 60, 120, 480 ;  $\dot{Q}'_{ig}$  (kW/m) : 10, 25, 50 or correspondingly  $x_{po}$  (m) : 0.2, 0.5, 1.0. Figure 8.1 - 8.3 show for the very thin material and low durations of the ignitor, the flame will never reach 5 m. But as these parameters are increased, propagation occurs and at faster speeds. Figure 8.4 shows the critical values of the parameters on propagation to 5 m. It is clear that all of these factors play a critical role in propagation.

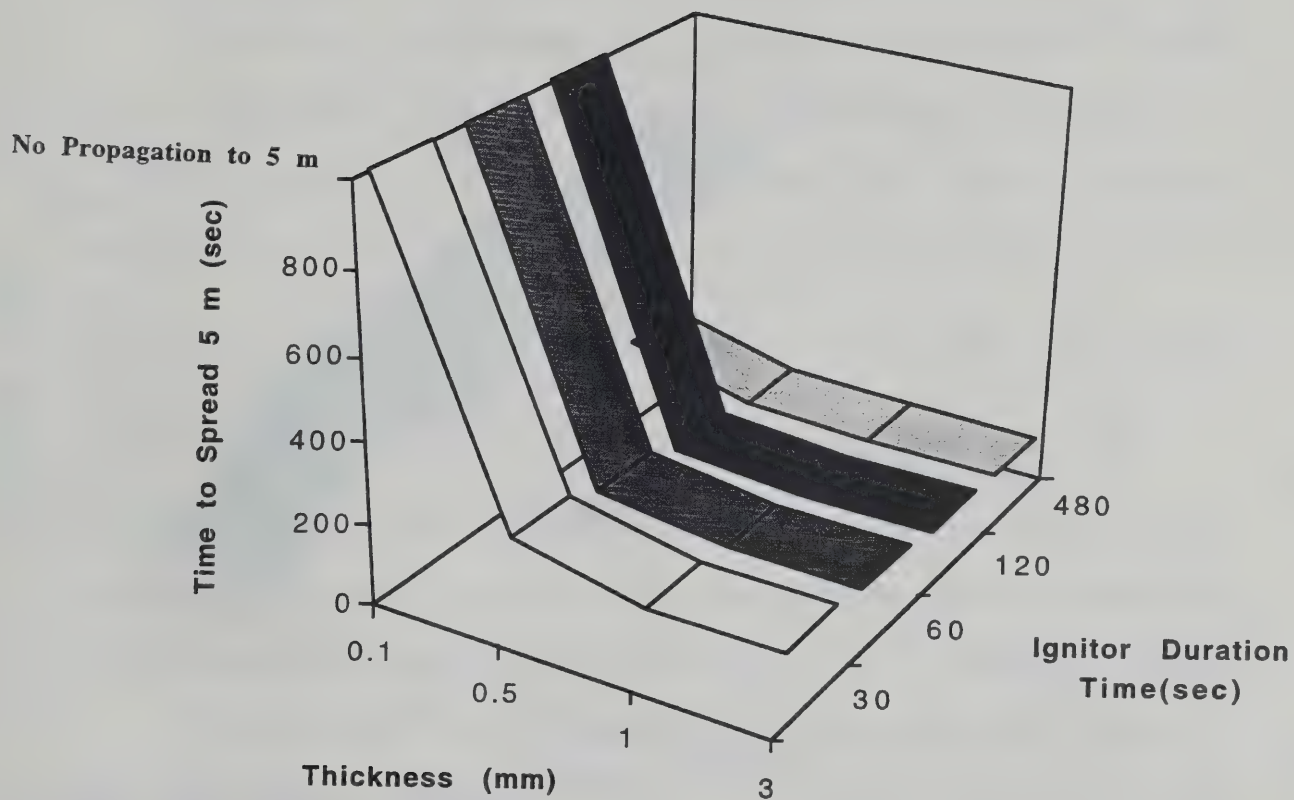


**FIGURE 8.1** Time to reach 5 m as a function of material thickness and ignitor duration at 10 kW/m for the ignitor.

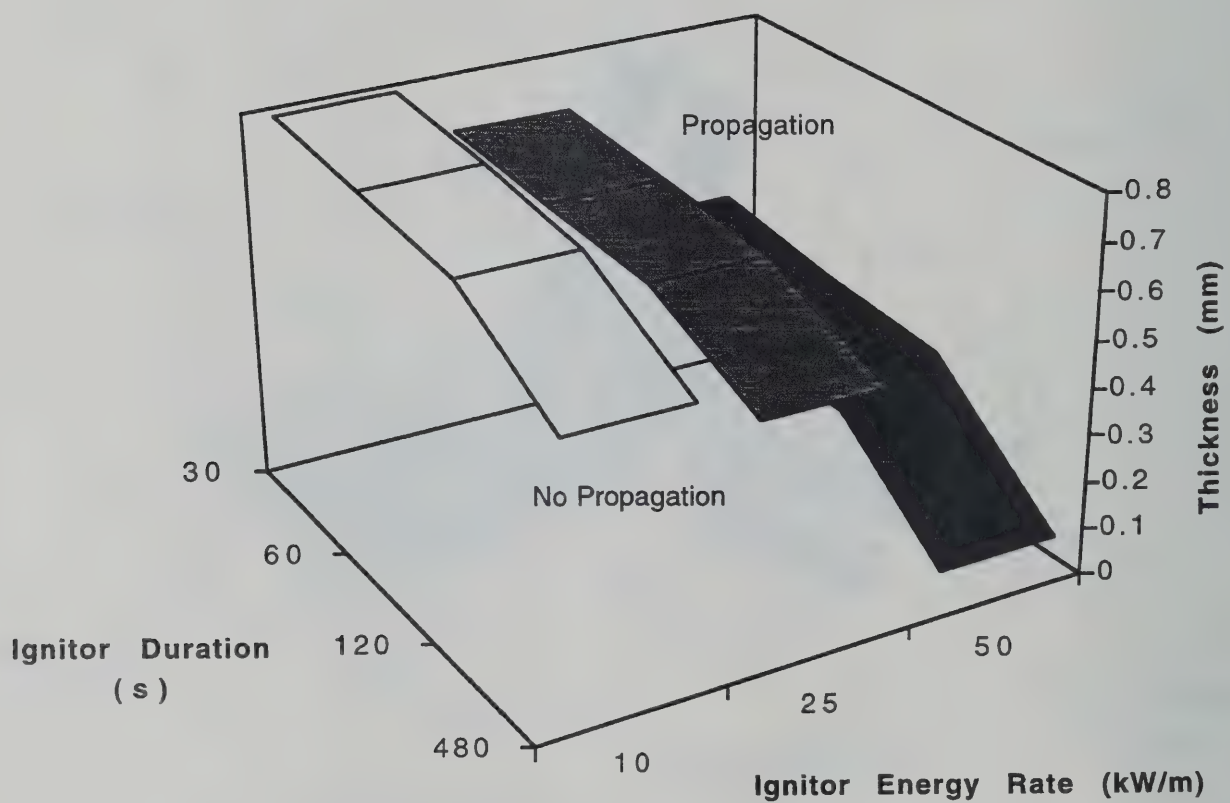


**FIGURE 8.2** Time to reach 5 m as a function of material thickness and ignitor duration at 25 kW/m for the ignitor.





**FIGURE 8.3** Time to reach 5 m as a function of material thickness and ignitor duration at 50 kW/m for the ignitor.



**FIGURE 8.4** Estimated critical values for propagation to 5 m.

Using the formulation outlined by Saito, Quintiere and Williams we developed a numerical algorithm that was checked with exact solutions for  $n=1$  and  $n=2/3$ .

We developed a general solution which included transient burning rate  $\dot{m}''(t)$  and used a specific function for thermoplastics, and included burnout and ignitor effects.

The comparisons illustrate the effect of model (that is,  $n=1$ ,  $2/3$  and  $\dot{m}''$  is constant) and value of  $k_p c$ . Flame height and flame velocity in case  $n=2/3$  have been found to be greater than those in case of  $n=1$  at initial, then these are switched. Also the effect of  $k_p c$  has been found that the bigger  $k_p c$ , the lower flame velocity. We also compared to experimental results, showing the general solution is in better agreement than the simpler analyses which assume  $\dot{m}''$  is constant. That is because the case of transient burning rate requires less energy than that of constant burning rate.

Future work should show a range of results from the general model to illustrate more clearly the role of the ignitor included duration time and heat flux and burnout related to thickness of material. It should be noted that any  $\dot{m}''(t)$  function can be used in the general algorithm.

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Properties	Name of Variable in Program	Data	Units
k	K	$0.432 \cdot 10^{-3}$	kW/m.k
$\rho$	DEN	1190.0	kg/m <sup>3</sup>
c	C	4.12	kJ/kg.k
T <sub>P</sub>	TP	375.0	°C
T <sub>a</sub>	TA	25.0	°C
$\dot{q}''_o$	QFLX0	25.0	kW/m <sup>2</sup>
q	QFLX	25.0	kW/m <sup>2</sup>
Q'	Q	0.0	kW/m
x <sub>po</sub>	XPO	0.3	m
$\dot{m}''$	CONST	5.4	g/m <sup>2</sup> .s
k <sub>f</sub>	KAY	0.01, N=1 0.067, N=2/3	m <sup>2</sup> /kW (m <sup>5</sup> /kW <sup>2</sup> ) <sup>1/3</sup>
t	I		sec

**TABLE A.1** The Variables and Data used for Testing





```

REAL KAY, QFLX,QFLX0, XPO, CONST, Q, K, DEN, C, TP, TA
REAL TAU, XP(0:1000), XF(0:1000), VP(0:1000), M(0:1000)
DATA KAY, QFLX, QFLX0,XPO, CONST, Q, N / 0.01,25.0,25.0,0.3,5.4,0,1/
DATA K, DEN, C, TP, TA, PI / 0.000432, 1190.0, 4.12, 375.0, 25.0,3.14/

OPEN(UNIT=11, FILE='CASE1.DAT', STATUS = 'UNKNOWN')

      TAU = ((PI*K*DEN*C)*(TP-TA)) / (4*QFLX0**2)
      DO 100 I=0,1000
          M(I) = CONST
          XP(I) = XPO*EXP((KAY*M(I)*QFLX-1)*I/TAU)
          XF(I) = KAY*((Q+(QFLX*M(I)*XP(I)))**N)
          VP(I) = (XF(I) -XP(I))/TAU
      WRITE(11,444) I, XP(I), XF(I), VP(I)

100      CONTINUE
444      FORMAT(10X, I5, 5X, E11.6, 5X, E11.6, 5X, E11.6)
      STOP
      END

```

**Program 1**    Exact Solution for n=1

```

REAL KAY, QFLX,QFLX0, XPO, CONST, Q, K, DEN, C, TP, TA
REAL TAU, XP(0:1000), XF(0:1000), VP(0:1000), M(0:1000)
DATA KAY,QFLX,QFLX0,XPO,CONST,Q,N /0.067,25.0,25.0,0.3,5.4,0.0,0.667/
DATA K, DEN, C, TP, TA, PI / 0.000432, 1190.0, 4.12, 375.0, 25.0, 3.14/

OPEN(UNIT=11, FILE='CASE2.DAT', STATUS = 'UNKNOWN')
  TAU = ((PI*K*DEN*C)*(TP-TA)) / (4*QFLX0**2)
  DO 100 I=0,1000
    M(I) = CONST
    A = EXP((-1*I)/(3*TAU))
    B = 1 - (XP0**(1./3.))/(KAY*((M(I)*QFLX)**(2./3.)))
    C = A*B
    D = (1 - C)**3
    E = (KAY**3)*(M(I)*QFLX)**2
    XP(I) = D*E
    XF(I) = KAY*((Q+(QFLX*M(I)*XP(I)))**N)
    VP(I) = (XF(I) -XP(I))/TAU
  WRITE(11,444) I, XP(I), XF(I), VP(I)
100  CONTINUE
444  FORMAT(1X,I5,1X,E11.6,1X,E11.6,1X,E11.6)
      STOP
      END

```

## Program 2 Exact Solution for $n=2/3$

```

PARAMETER(NMAX=1000, NAR=1000)
REAL KAY, QFLX,QFLX0, XPO, CONST, Q, K, DEN, C, TP, TA
REAL G1, G2, G3, N
REAL H, VP(0:NAR), M(0:NAR), XP(NAR), X(NMAX), SUM1(0:NAR), SUM2(0:NAR)
DATA KAY, QFLX, QFLX0,XPO, CONST, Q, N / 0.01,25.0,25.0,0.3,5.4,0,1/
DATA K, DEN, C, TP, TA, PI / 0.000432, 1190.0, 4.12, 375.0, 25.0, 3.14/
DATA TOL, H /0.0001,1 /

OPEN(UNIT=11, FILE='CASE3.DAT', STATUS = 'UNKNOWN')
  SUM1(0) = 0.0
  SUM2(0) = SUM1(0)
  TAU = ((PI*K*DEN*C)*(TP-TA)) / (4*QFLX0**2)
  VP(0) = (1/TAU)*((KAY*(Q+(QFLX*CONST*XP0))**N)-XP0)
  DO 100 I = 0,999
    M(I) = CONST
    M(I+1) = M(I)
    VP(I+1) = VP(I)
C ...   ITERATION LOOP

    K = 0
    X(1) = 0.0
10    K = K+1
    IF(K .GT.NMAX) GOTO 45
    G1 = M(I) * VP(I)
    G2 = M(I+1) * VP(I+1)
    G3 = VP(I) + VP (I+1)
    SUM1(I+1) = SUM1(I)+G1+G2
    SUM2(I+1) = SUM2(I)+G3

    VP(I+1) = (1/TAU)*(KAY*(Q+QFLX*(M(I+1)*XPO+
*          ((H/2)*(SUM1(I+1)+G1+G2))))**N)-(XPO+((H/2)*(SUM2(I+1)+G3))))
    X(K) = VP(I+1)

C ...   FIRST ITERATION. NO CONVERGENCE TEST
    IF(K .EQ.1) GOTO 10
    ERR = ABS((X(K) - X(K-1))/X(K))
    IF (ERR .LE. TOL) GOTO 50
    GOTO 10
45    WRITE(11,200)'DID NOT CONVERGE', VP(I+1), ERR
200    FORMAT(A16, E8.2, 5X, E8.2,)
50    WRITE(11,201)'CONVERGED AT', K,'-TH ITERATIONS WITH ERR=', ERR
201    FORMAT(A13, I5,A24,3X,E8.2)
    WRITE(11,444) I+1, VP(I+1)
100    CONTINUE
444    FORMAT(10X, I5, 5X, E11.6)
    STOP
  END

```

### Program 3 Numerical Solution for n=1

```

PARAMETER(NMAX=1000, NAR=1000)
REAL KAY, QFLX,QFLX0, XPO, CONST, Q, K, DEN, C, TP, TA
REAL G1, G2, G3, N
REAL H, VP(0:NAR), M(0:NAR), XP(NAR), X(NMAX), SUM1(0:NAR), SUM2(0:NAR)
DATA KAY, QFLX, QFLX0,XPO, CONST, Q, N / 0.067,25.0,25.0,0.3,5.4,0,0.667/
DATA K, DEN, C, TP, TA, PI / 0.000432, 1190.0, 4.12, 375.0, 25.0, 3.14/
DATA TOL, H /0.0001,1 /

OPEN(UNIT=11, FILE='CASE3.DAT', STATUS = 'UNKNOWN')
    SUM1(0) = 0.0
    SUM2(0) = SUM1(0)
    TAU = ((PI*K*DEN*C)*(TP-TA)) / (4*QFLX0**2)
    VP(0) = (1/TAU)*((KAY*(Q+(QFLX*CONST*XPO))**N)-XPO)
    DO 100 I = 0,999
        M(I) = CONST
        M(I+1) = M(I)
        VP(I+1) = VP(I)
C ...     ITERATION LOOP

        K = 0
        X(1) = 0.0
10      K = K+1
        IF(K .GT.NMAX) GOTO 45
        G1 = M(I) * VP(I)
        G2 = M(I+1) * VP(I+1)
        G3 = VP(I) + VP (I+1)
        SUM1(I+1) = SUM1(I)+G1+G2
        SUM2(I+1) = SUM2(I)+G3

        VP(I+1) = (1/TAU)*(KAY*(Q+QFLX*(M(I+1)*XPO+
*           ((H/2)*(SUM1(I+1)+G1+G2))))**N)-(XPO+((H/2)*(SUM2(I+1)+G3))))
        X(K) = VP(I+1)

C ...     FIRST ITERATION. NO CONVERGENCE TEST
        IF(K .EQ.1) GOTO 10
        ERR = ABS((X(K) - X(K-1))/X(K))
        IF (ERR .LE. TOL) GOTO 50
        GOTO 10
45      WRITE(11,200)'DID NOT CONVERGE', VP(I+1), ERR
200     FORMAT(A16, E8.2, 5X, E8.2,)
50      WRITE(11,201)'CONVERGED AT', K,'-TH ITERATIONS WITH ERR=', ERR
201     FORMAT(A13, I5,A24,3X,E8.2)
        WRITE(11,444) I+1, VP(I+1)
100     CONTINUE
444     FORMAT(10X, I5, 5X, E11.6)
        STOP
    END

```

#### Program 4 Exact Solution for $n=2/3$





## Material Data

$T_{\infty}$  - initial or ambient temperature

$k$  - thermal conductivity

$\rho$  - density

$c$  - specific heat

$T_{ig}$  - ignition Temperature

$\Delta H_v$  - heat of vaporization

$L$  - heat of gasification

$\Delta H_c$  - heat of combustion

$\alpha$  - thermal diffusivity

$\ell$  - thickness

$m''$  - burnable mass per unit area

Properties	Name of Variable in Program	Data	Units
$T_{\infty}$	AT	300	k
k	K	$0.432 \cdot 10^{-3}$	kW/m.k
$\rho$	DEN	1190.0	kg/m <sup>3</sup>
c	C	4.12	kJ/kg.k
$T_{ig}$	IGT	453.0	°K
$\Delta H_v$	HV	$L - C(T_{ig} - T_{\infty})$	kJ/kg
L	L	2770.0	kJ/kg
$\Delta H_c$	HC	25000.0	kJ/kg
$\alpha$	THDI	$k / \rho c$	m <sup>2</sup> /s
$\delta$	THI	0.005	m
m''	BM	$\rho l$	kg/m <sup>2</sup>

**TABLE C.1** Names of Variables and Data Used for Material

### Ignitor Characteristic

$\dot{Q}_{ig}$  - energy output rate

w - width of wall heated

$x_{fig}$  - ignitor flame height

$\Delta t_{ig}$  - duration of burning

$\dot{Q}'_{ig}$  - effective energy rate per wall width

Properties	Name of Variable in Program	Data	Units
$\dot{Q}_{ig}$	QIG	100.0	kW
w	W	0.5	m
$x_{fig}$	XFIG	$k_f(\dot{Q}_{ig}/w)$	m
$\Delta t_{ig}$	DELTIG	200.0	s
$\dot{Q}'_{ig}$	EQIG	$(x_{fig}/k_f)^{-n}$	kW/m

**TABLE C.2** Names of Variables and Data Used for Ignitor Characteristic

## Heat Flux

$\dot{q}''_f$  - heat flux from wall

$\dot{q}''_{fig}$  - heat flux from ignitor

Properties	Name of Variable in Program	Data	Units
$\dot{q}''_f$	QF	30.0	kW/m <sup>2</sup>
$\dot{q}''_{fig}$	QFIG	30.0	kW/m <sup>2</sup>

**TABLE C.3** Names of Variables and Data Used for Heat Flux

## Flame Height

$k_f$  - coefficient

$n$  - power

Properties	Name of Variable in Program	Data	Units
$k_f$	KF	0.01, $n=1$ 0.067, $n=2/3$	$m^2/kW$ $(m^5/kW^2)^{1/3}$
$n$	P	1 or $2/3$	

**TABLE C.4** Names of Variables and Data Used for Flame Height

### Computational Parameters

$h$  - time step

$\tau_{\max}$  - maximum time

$\varepsilon$  - tolerance for convergence

$\sigma$  - Stefan Boltzmann constant

Properties	Name of Variable in Program	Data	Units
$h$	H	1.0	s
$\tau_{\max}$	TAUMAX	1000.0	s
$\varepsilon$	TOL	0.05	
$\sigma$	SIG	$5.67 \cdot 10^{-11}$	$\text{kW/m}^2\text{k}^4$

**TABLE C.5** Names of Variables and Data Used for Computational Parameters



## Computed Parameters

$t_{ig}$  - ignition time

$$t_{ig} = \frac{\pi}{4} k\rho c \left( \frac{T_{ig} - T_{\infty}}{\dot{q}''_{fig}} \right)^2$$

$\Delta t_f$  - spread time

$$\Delta t_f = \frac{\pi}{4} k\rho c \left( \frac{T_{ig} - T_{\infty}}{\dot{q}''_f} \right)^2$$

$\delta_s$  - steady penetration depth

$$\delta_s = \frac{2kL}{c(\dot{q}''_f - \sigma T_{ig}^4)}$$

$\dot{q}''$  - net flame heat flux

$$\dot{q}'' = \dot{q}''_f - \sigma T_{ig}^4$$

$\tau^*$  - burn time constant

$$\tau^* = \frac{\delta_s^2}{6\alpha} \frac{\Delta H_v}{L}$$

Properties	Name of Variable in Program	Units
$t_{ig}$	TIG	s
$\Delta t_f$	TF	s
$\delta_s$	DS	m
$\dot{q}''$	NETQ	kW/m <sub>2</sub>
$\tau^*$	TAUS	s

**TABLE C.6** Names of Variables Used for Computed Parameters

## Output Variables

$\tau(i+1)$  - time after ignition

$\tau_f(i)$  - time of arrival of flame tip at position  $x=x_p(i)$

$\dot{m}''(j,i)$  - burning rate per unit area at time  $\tau(j)$  and position  $x=x_p(i)$

$\tau_b(i)$  - burnout time at  $x=x_p(i)$

$x_b(i)$  - burnout position at  $\tau=\tau(i)$

$\dot{Q}(i)$  - total energy release rate

$x_p(i)$  - pyrolysis front position

$x_f(i)$  - flame tip position

$V_p(i)$  - velocity of the pyrolysis front

Properties	Name of Variable in Program	Units
$\tau(i+1)$	TAU(I+1)	s
$\tau_f(i)$	TAUF(I)	s
$\dot{m}''(j,i)$	M(J,I)	$g/m^2.s$
$\tau_b(i)$	TAUB(I)	s
$x_b(i)$	XB(I)	m
$\dot{Q}(i)$	TQ(I)	kW
$x_p(i)$	XP(I)	m
$x_f(i)$	XF(I)	m
$V_p(i)$	VP(I)	m/s

**TABLE C.7** Names of Variables Used for output



PARAMETER(NMAX=1005)

C ...     DECLATION PART

REAL AT  
DATA AT /20.0/

C ...     MATERIAL DATA

REAL K1, DEN, C, IGT, HV, L, HC, THDI, THI, BM  
DATA K1, DEN, C, IGT, L /0.000432,1190.0,4.12,180.0,2770.0/  
DATA HC, THI /25000.0,0.005/

C ...     IGNITOR CHARACTERISTICS

REAL QIG, W, XFIG, DELTIG, EQIG  
DATA QIG, W, DELTIG /100.0,0.5,200.0/

C ...     HEAT FLUX

REAL QF, QFIG  
DATA QF, QFIG /30.0,30.0/

C ...     FLAME HEIGHT

REAL KF, P  
DATA KF, P /0.01,1.0/

C ...     COMPUTATIONAL PARAMETERS

REAL H, TAUMAX, SIG  
DATA H, SIG, TAUMAX /1.0,5.67E-11,1000.0/

C ...     COMPUTED PARAMETERS

REAL TIG, TF, DS, NETQ, TAUS, DELIG, DEL1

C ...     MAXIMUM NUMBER OF STEPS, N

REAL N

C ...     FOR ITERATION

REAL TOL  
DATA TOL /0.05/



C ...      VARIABLES

```
REAL TAU(2000), TAUF(NMAX), TAUP(NMAX), TAUB(NMAX)
REAL XP(NMAX), XF(NMAX), XB(NMAX)
REAL M, VP(NMAX)
REAL Q1, Q2, TQ(NMAX)
REAL SUM1(0:2000), SUM2(NMAX)
INTEGER I
```

C ...      COMMON STATEMENT

```
COMMON /TIME/ TAU, TAUP, TAUF
COMMON /BUNT/ TAUB
COMMON /HEIGHT/ XB, XP, XF, VP
COMMON /BRT/ BM, SUM1
COMMON /DELTA/ DEL1
COMMON /RTM/ DS, TOL, NETQ, IGT, AT, HV, TAUS, THDI
COMMON /SPD/ DELTIG, TIG, XFIG, HC, SUM2, KF, P, TF, TQ, W
COMMON /CONT/ H, K1
```

```
OPEN(UNIT=11, FILE="TEST3.DAT", STATUS = 'UNKNOWN')
```

C...      INITIALIZATION PART

C ...      MATERIAL DATA

```
HV = L - (C*(IGT - AT))
THDI = (K1) / (DEN * C)
BM = DEN*THI
```

C ...      FLAME HEIGHT

```
XFIG = KF * (QIG/W)**P
EQIG = (XFIG/KF)**(1/P)
```

C ...      COMPUTED PARAMETERS

```
TIG = (3.14159/4) * (K1*DEN*C) * ((IGT-AT)/QFIG)**2
TF = (3.14159) * (K1*DEN*C) * ((IGT-AT)/QF)**2
DS = (2*K1*L) / (C * (QF - (SIG*IGT**4)))
NETQ = QF - (SIG*IGT**4)
TAUS = (DS**2*HV) / (6*THDI*L)
```

C ...      MAXIMUM NUMBER OF STEP

```
N = TAUMAX / H
```

C ...

C ...     **INITIAL CONDITION**

SUM1(0) = 0.0

SUM2(1) = 0.0

I = 1

TAU(I) = 0.0

TAUP(I) = 0.0

TAUF(I) = -TIG

C ...     DEL1 IS DEL IN THE INITIAL CONDITION.

DELIG = SQRT((6\*THDI) \* (TAU(I) - TAUF(I)))

DEL1 = DELIG

M = (NETQ - (((2\*K1)\*(IGT-AT)) / (DEL1))) / HV

C ...     **FIND BURNOUT TIME (TAUB)**

CALL BURNOUT(I)

IF((TAUB(1) .GE. TAU(I)) .OR. ((DELTIG-TIG) .GE. TAU(I))) GOTO 10

EQIG = 0.0

10     EQIG = EQIG

Q1 = HC \* M

XB(I) = 0.0

XF(I) = XB(I) + KF \* (EQIG + Q1)

XP(I) = XFIG

VP(I) = (XF(I) - XP(I)) / TF

C ...     **TOTAL ENERGY RELEASE RATE**

TQ(I) = (EQIG + Q1) \* W

WRITE(11,20) TAU(I),',',XB(I),',',XP(I),',',XF(I),',',VP(I),',', TQ(I)

20     FORMAT(F7.2,A2,1X,E11.6, A2,1X,E11.6,A2,1X,E11.6, A2,1X,E11.6,  
A2,1X,E11.6)

```

C ...
C ...
C ...   MAIN LOOP

      DO 30 I = 1, N

            TAU(I+1) = TAU(I) + H

            IF (XP(I) .LE. XF(1)) THEN
            TAUF(I+1) = 0.0
            ELSE
            TAUF(I+1) = TAUF(I)
            END IF

C ...   FIND BURNOUT TIME

            CALL BURNOUT (I+1)

C ...   FIND BURNOUT POSITION

            IF (TAU(I+1) .GE. TAUB(1)) THEN
            CALL SEARCHB(I+1)
            ELSE
            XB(I+1) = 0.0
            END IF

C ...   FIND SPREAD RESULT

            CALL SPREAD(I+1)

C ...   FIND TIME FLAME TIP REACHED FIRST

            IF (XP(I+1) .LE. XF(1)) THEN
            TAUF(I+1) = 0.0
            ELSE
            CALL SEARCHF(I+1)
            END IF

30      CONTINUE

      STOP
      END

```

```

C ...
C ...    FIND BURNOUT TIME

SUBROUTINE BURNOUT(J)

PARAMETER(NMAX=1005)

REAL TAU(2000), TAUP(NMAX), TAUF(NMAX)
REAL TAUB(NMAX)
REAL BM, SUM1(0:2000)
REAL DS, TOL, NETQ, IGT, AT, HV, TAUS, THDI
REAL H, K1
REAL A, B, G1, G2, G3
INTEGER I, J, N1

COMMON /TIME/ TAU, TAUP, TAUF
COMMON /BUNT/ TAUB
COMMON /BRT/ BM, SUM1
COMMON /RTM/ DS, TOL, NETQ, IGT, AT, HV, TAUS, THDI
COMMON /CONT/ H, K1

      IF(J .EQ. 1) THEN
        TAUP(J) = 0.0
      ELSE
        I = J - 1
        TAUP(J) = TAU(I+1)
      END IF

      K = 0
999      K = K+1
          CALL ROOTM(K, J, A)

C ...  A = DEL AT K AND J, B = DEL AT K+1, J
C ...    G1 = M(K,J), G2 = M(K+1,J)

          G1 = (NETQ - (((2*K1)*(IGT-AT)) / A)) / HV

          IF(G1 .LT. 0) THEN
            G1 = 0.0
          ELSE
            G1 = G1
          END IF

          CALL ROOTM(K+1, J, B)

          G2 = (NETQ - (((2*K1)*(IGT-AT)) / B)) / HV

          IF(G2 .LT. 0) THEN
            G2 = 0.0
          ELSE
            G2 = G2

```

```

END IF

SUM1(K) = SUM1(K-1) + G1 + G2
G3 = (H/2) * SUM1(K)

IF(G3 .GE. BM) THEN
N1 = K + 1
TAUB(J) = (N1 - 1) * H
ELSE
GOTO 999
END IF

RETURN
END

```

```

C ...
C ...    FIND THERMAL PENETRATION DEPTH

SUBROUTINE ROOTM(I, J, DEL3)

PARAMETER(NMAX=1005)

REAL TAU(2000), TAUP(NMAX), TAUF(NMAX)
REAL DS, TOL, NETQ, IGT, AT, HV, TAUS, THDI
REAL DEL3, ERR, DELIG, DEL1, X(0:305)
INTEGER I, J, K

COMMON /TIME/ TAU, TAUP, TAUF
COMMON /DELTA/ DEL1
COMMON /RTM/ DS, TOL, NETQ, IGT, AT, HV, TAUS, THDI
COMMON /CONT/ H, K1

        IF(I .EQ. 1) THEN
            TAU(I) = 0.0
        ELSE
            TAU(I) = TAU(I-1) + H
        END IF

        DELIG = SQRT((6*THDI) * (TAUP(J) - TAUF(J)))
        X(0) = DEL1

C ...    ITERATION LOOP TO FIND DELTA

C ...    X(K) IS DELTA

        DO 50 K = 1, 300

            IF(K .GT. 300) GOTO 70

            X(K) = DS - ((DS - DELIG) * EXP(((DELIG - X(K-1))/DS) - ((TAU(I) -
                TAUP(J))/TAUS)))

            IF(X(K) .LT. 0) THEN
                X(K) = X(K-1)
                GOTO 60
            ELSE
                X(K) = X(K)
            END IF

            ERR = ABS((X(K) - X(K-1)) / X(K))

            IF(ERR .LE. TOL) GOTO 60
50          CONTINUE
70          WRITE(11,80) 'DID NOT CONVERGE', X(K), ERR
60          DEL3 = X(K)
80          FORMAT(A16, 2X, E11.6, 5X, E11.6)

```

RETURN  
END



```

C ...
C ...    FIND J SUCH THAT TAUB(J) = TAU(I)

SUBROUTINE SEARCHB(I)

PARAMETER(NMAX=1005)

REAL TAU(2000), TAUP(NMAX), TAUF(NMAX), TAUB(NMAX)
REAL XB(NMAX), XP(NMAX), XF(NMAX), VP(NMAX)
INTEGER I, J

COMMON /TIME/ TAU, TAUP, TAUF
COMMON /BUNT/ TAUB
COMMON /HEIGHT/ XB, XP, XF, VP

        DO 90 J = 1, I

        IF(TAUB(J) .GE. TAU(I)) GOTO 700

90      CONTINUE

700     XB(I) = XP(J)

        RETURN
        END

```

```

C ...
C ...      FIND SPREAD DATA

SUBROUTINE SPREAD(J)

PARAMETER(NMAX=1005)

REAL TAU(2000), TAUP(NMAX), TAUF(NMAX), TAUB(NMAX)
REAL XB(NMAX), XP(NMAX), XF(NMAX), VP(NMAX)
REAL DS, TOL, NETQ, IGT, AT, HV, TAUS, THDI
REAL DELTIG, TIG, XFIG, HC, SUM2(NMAX), SUM3(0:NMAX), KF, P, TF,
REAL TQ(NMAX), W
REAL X(0:1005), H, K1, SUM4(0:NMAX), D, E, F, G
REAL EQIG, ERR1, A, B, C, M1, M2, M3, M4, M5, FI(1005), FI1(1005), Q1, Q2
INTEGER I, J, K

COMMON /TIME/ TAU, TAUP, TAUF
COMMON /BUNT/ TAUB
COMMON /HEIGHT/ XB, XP, XF, VP
COMMON /RTM/ DS, TOL, NETQ, IGT, AT, HV, TAUS, THDI
COMMON /SPD/ DELTIG, TIG, XFIG, HC, SUM2, KF, P, TF, TQ, W
COMMON /CONT/ H, K1

      SUM4(0) = 0.0
      SUM3(0) = 0.0

C ...      FIND EFFECTIVE ENERGY RATE PER WALL WIDTH

      IF(((DELTIG - TIG) .GE. TAU(J)) .OR. (TAUB(1) .GE. TAU(J))) THEN
      EQIG = (XFIG / KF) ** (1/P)
      ELSE
      EQIG = 0.0
      END IF

C ...      FIND Q1

C ...      M1 = M(J,1)

      CALL ROOTM(J, 1, A)

      IF(TAUB(1) .LT. TAU(J)) THEN
      M1 = 0.0
      ELSE
      M1 = (NETQ - (((2*K1)*(IGT-AT)) / A)) / HV
      END IF

      IF(M1 .LT. 0) THEN
      M1 = 0.0
      ELSE
      M1 = M1
      END IF

```

```

      Q1 = HC * M1 * XFIG

C ...   ITERATION LOOP TO FIND VP

      VP(J) = VP(J-1)

      DO 100 K = 1, 1000

      IF (K .GT. 1) GOTO 102
      IF (K .GE. 1000) GOTO 110
      IF (J-2 .EQ. 0) GOTO 102

C ...   FIND Q2

      DO 101 I = 1, J-2

C ...   M2 = M(J, J-1), M3 = (MJ, J)

      CALL ROOTM(J, I, B)

      IF(TAUB(I) .LT. TAU(J)) THEN
      M2 = 0.0
      ELSE
      M2 = (NETQ - (((2*K1)*(IGT-AT)) / B)) / HV
      END IF

      IF(M2 .LT. 0) THEN
      M2 = 0.0
      ELSE
      M2 = M2
      END IF

      D = M2 * VP(I)

      CALL ROOTM(J, I+1, C)

      IF(TAUB(I+1) .LT. TAU(J)) THEN
      M3 = 0.0
      ELSE
      M3 = (NETQ - (((2*K1)*(IGT-AT)) / C)) / HV
      END IF

      IF(M3 .LT. 0) THEN
      M3 = 0.0
      ELSE
      M3 = M3
      END IF

      E = M3 * VP(I+1)

```

```

SUM3(I) = SUM3(I-1) + VP(I) + VP(I+1)
SUM4(I) = SUM4(I-1) + D + E

IF(I .EQ. J-2) GOTO 102

101    CONTINUE

102    CALL ROOTM(J, J-1, F)

        IF(TAUB(J-1) .LT. TAU(J)) THEN
            M4 = 0.0
        ELSE
            M4 = (NETQ - (((2*K1)*(IGT-AT)) / F)) / HV
        END IF

        IF(M4 .LT. 0) THEN
            M4 = 0.0
        ELSE
            M4 = M4
        END IF

        CALL ROOTM(J, J, G)

        IF(TAUB(J) .LT. TAU(J)) THEN
            M5 = 0.0
        ELSE
            M5 = (NETQ - (((2*K1)*(IGT-AT)) / G)) / HV
        END IF

        IF(M5 .LT. 0) THEN
            M5 = 0.0
        ELSE
            M5 = M5
        END IF

        IF(J-2 .EQ. 0) THEN
            SUM3(J-2) = 0.0
            SUM4(J-2) = 0.0
        ELSE
            SUM3(J-2) = SUM3(J-2)
            SUM4(J-2) = SUM4(J-2)
        END IF

        FI(K) = (H/2) * (SUM4(J-2) + (F*VP(J-1)) + (G*VP(J)))
        Q2 = HC * FI(K)

C ...    FIND FLAME HEIGHT(XF)

            XF(J) = ((KF) * ((EQIG + Q1 + Q2)**P)) + XB(J)

C ...    FIND XP

```

```

        FI1(K) = (H/2) * (SUM3(J-2) + VP(J))
        XP(J) = XFIG + FI1(K)

C ...    FIND VP

        VP(J) = (XF(J) - XP(J)) / TF
        X(K) = VP(J)
        ERR1 = ABS((X(K) - X(K-1)) / X(K))
        IF (ERR1 .LE. TOL) GOTO 120
100      CONTINUE
110      WRITE(11,130) 'DID NOT CONVERGE', X(K), ERR1
130      FORMAT(A16, 2X, E11.6, 5X, E11.6)
120      TQ(J) = (EQIG + Q1 + Q2) * W

        WRITE(11,150)TAU(J),',',XB(J),',',XP(J),',',XF(J),',',VP(J),',',TQ(J)

150      FORMAT(F7.2,A2,1X,E11.6,A2,1X,E11.6,A2,1X,E11.6, A2,1X,E11.6,
        A2,1X,E11.6)

        RETURN
        END

```

C ...  
C ... FIND THE TIME THE FLAME TIP FIRST REACHED X=XP(J)

SUBROUTINE SEARCHF(J)

PARAMETER(NMAX=1005)

REAL TAU(2000), TAUP(NMAX), TAUF(NMAX)  
REAL XB(NMAX), XP(NMAX), XF(NMAX), VP(NMAX)  
REAL H  
INTEGER J, K

COMMON /TIME/ TAU, TAUP, TAUF  
COMMON /HEIGHT/ XB, XP, XF, VP  
COMMON /CONT/ H

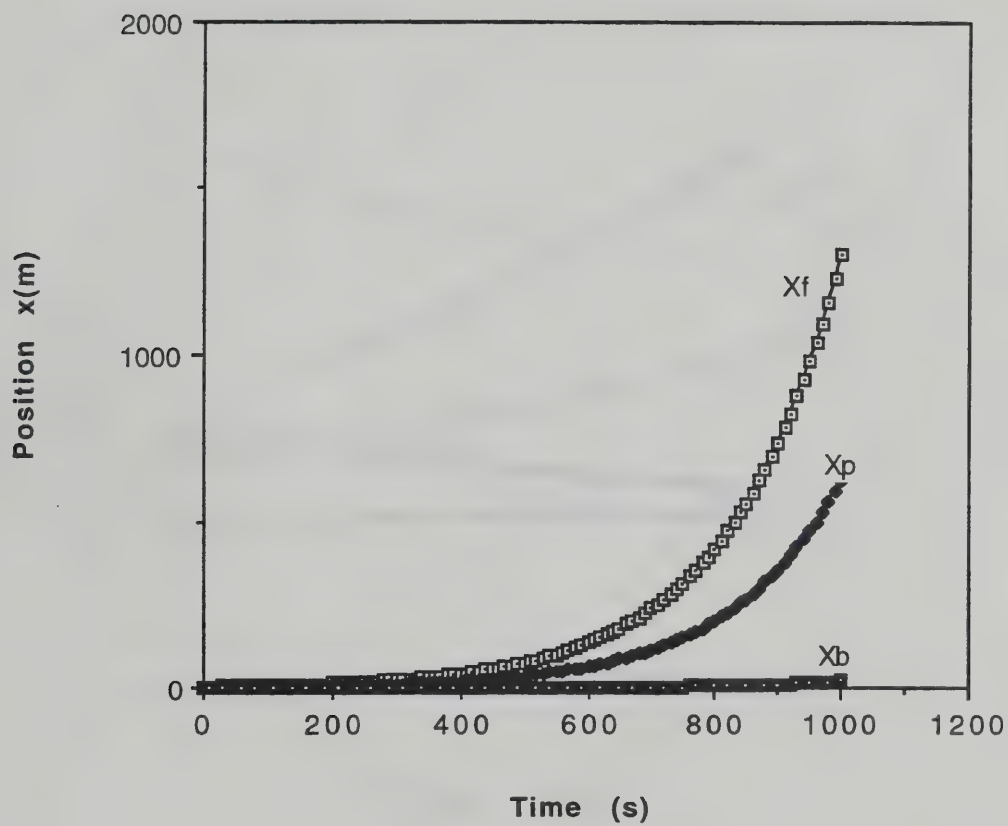
160 K = 0  
K = K+1

IF(XF(K) .GT. XP(J)) THEN  
TAUF(J) = (K - 1) \* H  
ELSE  
TAUF(J) = 0.0  
GOTO 160  
END IF

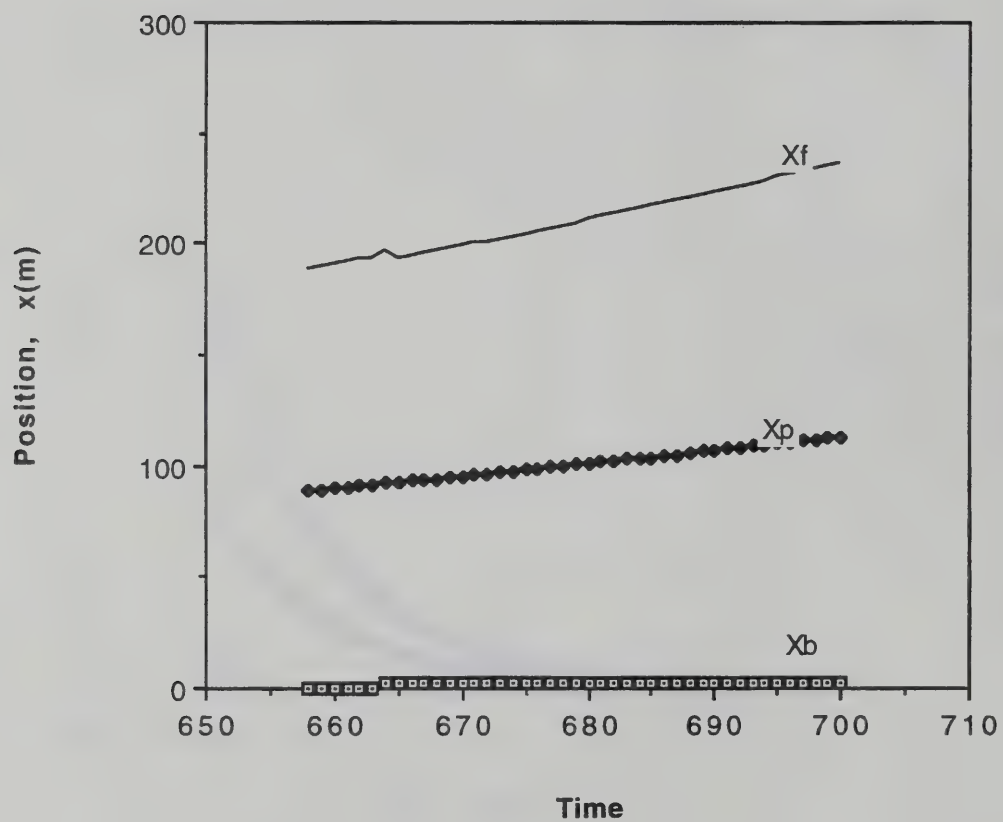
RETURN  
END



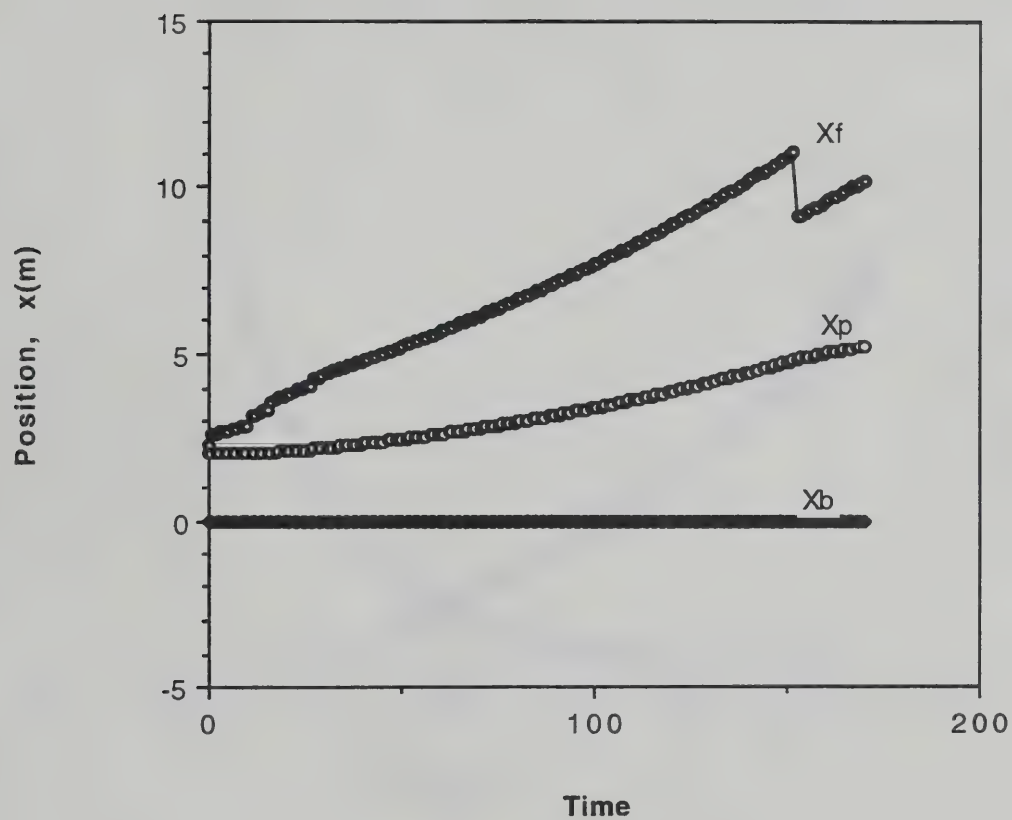




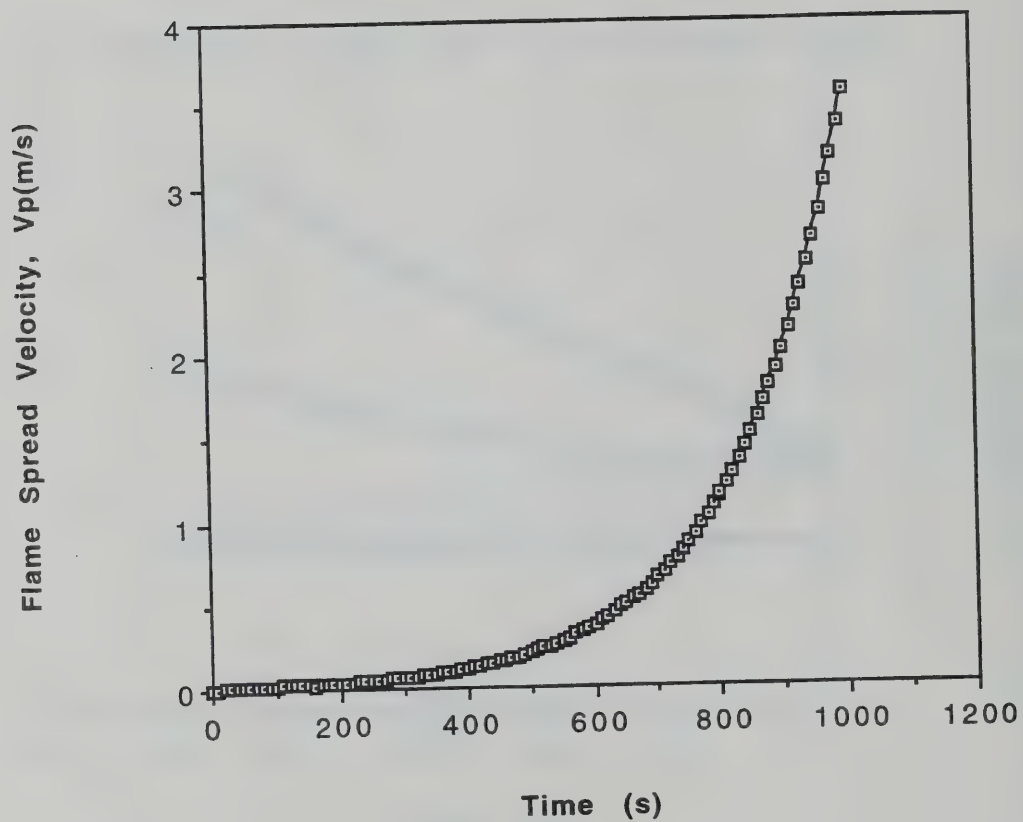
**FIGURE E.1** Flame tip position, pyrolysis front position, and burnout position as a function of time of generalized flame spread model for PMMA.



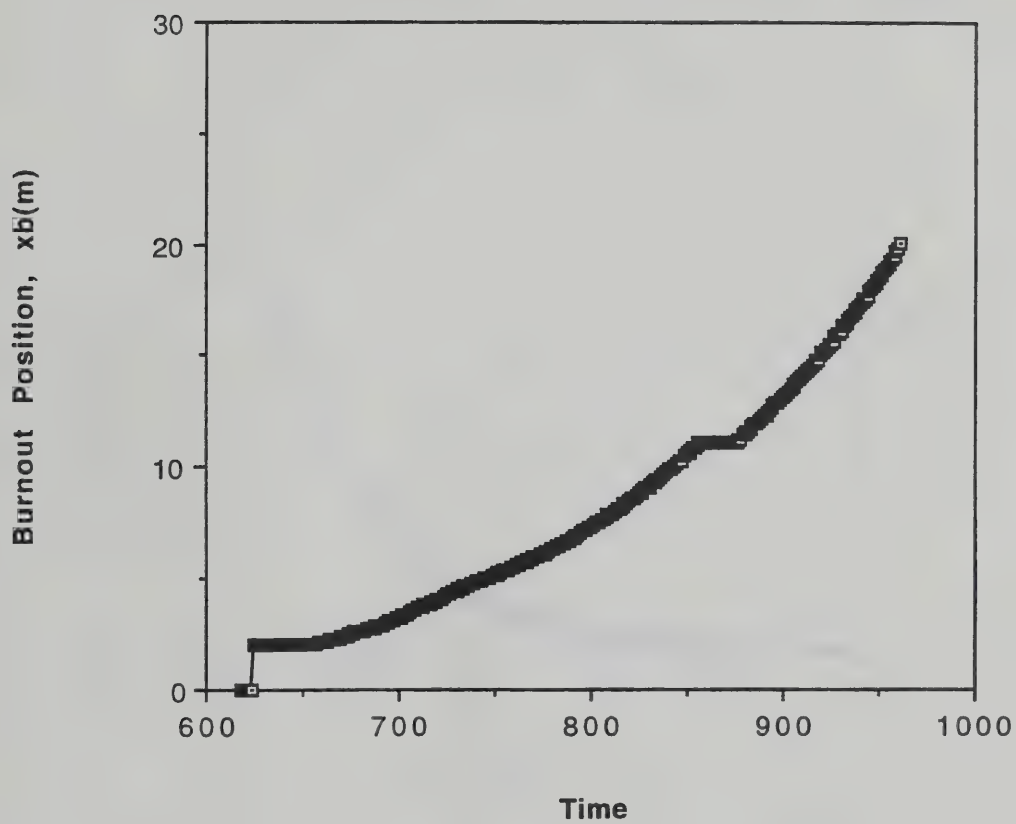
**FIGURE E.2** Burnout effect of Flame tip position as a function of time of generalized flame spread model for PMMA.



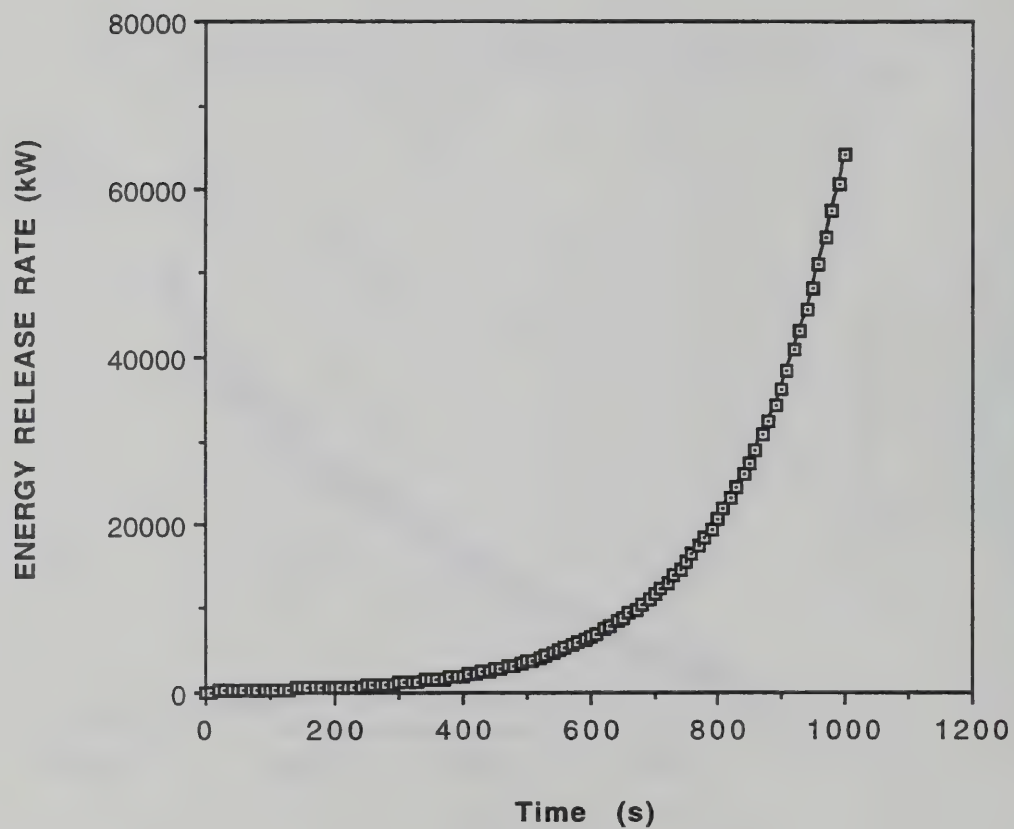
**FIGURE E.3** Ignitor effect of Flame tip position as a function of time of generalized flame spread model for PMMA.



**FIGURE E.4** Velocity of the pyrolysis front as a function of time of generalized flame spread model for PMMA.



**FIGURE E.5** Burnout position as a function of time of generalized flame spread model for PMMA.



**FIGURE E.6** Energy release rate as a function of time of generalized flame spread model for PMMA.





Properties	Name of Variable in Program	Data	Units
k	K	$2.63 \times 10^{-4}$	kW/m.°C
$\rho$	DEN	1190.0	kg/m <sup>3</sup>
c	C	2.09	kJ/kg.°C
T <sub>P</sub>	TP	363.0	°C
T <sub>a</sub>	TA	20.0	°C
$\dot{q}''_o$	QFLX0	25.0	kW/m <sup>2</sup>
q	QFLX	25.0	kW/m <sup>2</sup>
Q'	Q	0.0	kW/m
x <sub>po</sub>	XPO	0.02	m
$\dot{m}''$	CONST	9.6	g/m <sup>2</sup> .s
k <sub>f</sub>	KAY	0.01, N=1 0.067, N=2/3	m <sup>2</sup> /kW (m <sup>5</sup> /kW <sup>2</sup> ) <sup>1/3</sup>
t	I		sec

TABLE F.1 Orloff, de Ris, and Markstein's Data for Comparison

Properties	Name of Variable in Program	Data	Units
k	K	$0.346 \cdot 10^{-3}$	kW/m.°C
$\rho$	DEN	1180.0	kg/m <sup>3</sup>
c	C	2.5	kJ/kg.°C
T <sub>P</sub>	TP	363.0	°C
T <sub>a</sub>	TA	20.0	°C
$\dot{q}''_o$	QFLX0	25.0	kW/m <sup>2</sup>
q	QFLX	25.0	kW/m <sup>2</sup>
Q'	Q	0.0	kW/m
x <sub>po</sub>	XPO	0.02	m
$\dot{m}''$	CONST	9.6	g/m <sup>2</sup> .s
k <sub>f</sub>	KAY	0.01, N=1 0.067, N=2/3	m <sup>2</sup> /kW (m <sup>5</sup> /kW <sup>2</sup> ) <sup>1/3</sup>
t	I		sec

**TABLE F.2** LIFT's Data for Comparison



```

REAL KAY, QFLX,QFLX0, XPO, CONST, Q, K, DEN, C, TP, TA
REAL TAU, XP(0:1000), XF(0:1000), VP(0:1000), M(0:1000)
DATA KAY, QFLX, QFLX0,XPO, CONST, Q, N / 0.01,25.0,25.0,0.02,9.6,0,1/
DATA K, DEN, C, TP, TA, PI / 0.000263, 1190.0, 2.09, 363.0, 20.0,3.14/

OPEN(UNIT=11, FILE='CASE1.DAT', STATUS = 'UNKNOWN')

      TAU = ((PI*K*DEN*C)*(TP-TA)) / (4*QFLX0**2)
      DO 100 I=0,1000
            M(I) = CONST
            XP(I) = XPO*EXP((KAY*M(I)*QFLX-1)*I/TAU)
            XF(I) = KAY*((Q+(QFLX*M(I)*XP(I)))**N)
            VP(I) = (XF(I) -XP(I))/TAU
      WRITE(11,444) I, XP(I), XF(I), VP(I)

100      CONTINUE
444      FORMAT(10X, I5, 5X, E11.6, 5X, E11.6, 5X, E11.6)
      STOP
      END

```

**Program 1** Exact Solution for  $n=1$  used Orloff, de Ris, and Markstein's Data

```

REAL KAY, QFLX,QFLX0, XPO, CONST, Q, K, DEN, C, TP, TA
REAL TAU, XP(0:1000), XF(0:1000), VP(0:1000), M(0:1000)
DATA KAY, QFLX, QFLX0,XPO, CONST, Q, N / 0.067,25.0,25.0,0.02,9.6,0,0.667/
DATA K, DEN, C, TP, TA, PI / 0.000263, 1190.0, 2.09, 363.0, 20.0,3.14/
OPEN(UNIT=11, FILE='CASE2.DAT', STATUS = 'UNKNOWN')
    TAU = ((PI*K*DEN*C)*(TP-TA)) / (4*QFLX0**2)
    DO 100 I=0,1000
        M(I) = CONST
        A = EXP((-1*I)/(3*TAU))
        B = 1 - (XP0**(1./3.))/(KAY*((M(I)*QFLX)**(2./3.)))
        C = A*B
        D = (1 - C)**3
        E = (KAY**3)*(M(I)*QFLX)**2
        XP(I) = D*E
        XF(I) = KAY*((Q+(QFLX*M(I)*XP(I)))**N)
        VP(I) = (XF(I) -XP(I))/TAU
    WRITE(11,444) I, XP(I), XF(I), VP(I)
100    CONTINUE
444    FORMAT(1X,I5,1X,E11.6,1X,E11.6,1X,E11.6)
    STOP
    END

```

**Program 2** Exact Solution for  $n=2/3$  used Orloff, de Ris, and Markstein's Data

```

REAL KAY, QFLX,QFLX0, XPO, CONST, Q, K, DEN, C, TP, TA
REAL TAU, XP(0:1000), XF(0:1000), VP(0:1000), M(0:1000)
DATA KAY, QFLX, QFLX0,XPO, CONST, Q, N / 0.01,25.0,25.0,0.02,9.6,0,1/
DATA K, DEN, C, TP, TA, PI / 0.000346, 1180.0, 2.5, 363.0, 20.0,3.14/

OPEN(UNIT=11, FILE='CASE1.DAT', STATUS = 'UNKNOWN')

      TAU = ((PI*K*DEN*C)*(TP-TA)) / (4*QFLX0**2)
      DO 100 I=0,1000
          M(I) = CONST
          XP(I) = XPO*EXP((KAY*M(I)*QFLX-1)*I/TAU)
          XF(I) = KAY*((Q+(QFLX*M(I)*XP(I)))**N)
          VP(I) = (XF(I) -XP(I))/TAU
      WRITE(11,444) I, XP(I), XF(I), VP(I)

100      CONTINUE
444      FORMAT(10X, I5, 5X, E11.6, 5X, E11.6, 5X, E11.6)
      STOP
      END

```

**Program 3**    Exact Solution for n=1 used LIFT Data

```

REAL KAY, QFLX,QFLX0, XPO, CONST, Q, K, DEN, C, TP, TA
REAL TAU, XP(0:1000), XF(0:1000), VP(0:1000), M(0:1000)
DATA KAY, QFLX, QFLX0,XPO, CONST, Q, N / 0.067,25.0,25.0,0.02,9.6,0,0.667/
DATA K, DEN, C, TP, TA, PI / 0.000346, 1180.0, 2.5, 363.0, 20.0,3.14/
OPEN(UNIT=11, FILE='CASE2.DAT', STATUS = 'UNKNOWN')
    TAU = ((PI*K*DEN*C)*(TP-TA)) / (4*QFLX0**2)
    DO 100 I=0,1000
        M(I) = CONST
        A = EXP((-1*I)/(3*TAU))
        B = 1 - (XP0**(1./3.))/(KAY*((M(I)*QFLX)**(2./3.)))
        C = A*B
        D = (1 - C)**3
        E = (KAY**3)*(M(I)*QFLX)**2
        XP(I) = D*E
        XF(I) = KAY*((Q+(QFLX*M(I)*XP(I)))**N)
        VP(I) = (XF(I) -XP(I))/TAU
    WRITE(11,444) I, XP(I), XF(I), VP(I)
100    CONTINUE
444    FORMAT(1X,I5,1X,E11.6,1X,E11.6,1X,E11.6)
    STOP
    END

```

#### Program 4 Exact Solution for $n=2/3$ used LIFT Data



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